

Physics-constrained Deep learning of nonlinear normal modes of spatiotemporal fluid flow dynamics

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Abstract. This study presents a physics-constrained deep learning method for identifying and visualizing the invariant nonlinear normal modes (NNMs), which contain the spatiotemporal dynamics of fluid flow potentially exhibiting strong nonlinearity. To develop the nonlinear modal transformation, NNM-CNN-AE integrates a multi-temporal-step dynamics prediction block with a convolutional autoencoder constrained by NNM physics (NNM-CNN-AE). In addition, we simultaneously learn the NNMs containing the spatiotemporal dynamics of the flow fields, reduced-order reconstruction and predict long-term flow fields.

Introduction

In this study, we present a physics-constrained deep learning method to discover and visualize from data the invariant nonlinear normal modes (NNMs) which contain the spatiotemporal dynamics of the fluid flow potentially containing strong nonlinearity. Specifically, we develop a NNM-physics-constrained convolutional autoencoder (NNM-CNN-AE) integrated with a multi-temporal-step dynamics prediction block to learn the nonlinear modal transformation, the NNMs containing the spatiotemporal dynamics of the flow, and reduced-order reconstruction and long-time future-state prediction of the flow fields, simultaneously. In test cases, we apply the developed method to analyze different flow regimes past a cylinder, including laminar flows with Low Reynolds Number (LRN) in transient and steady states ($R_D=100$) and High Reynolds Number (HRN) flow ($R_D=1000$), respectively. The results indicate that the identified NNMs are able to reveal the nonlinear spatiotemporal dynamics of these flows, and the NNMs-based reduced-order modeling consistently achieves better accuracy with orders of magnitudes smaller errors in construction and prediction of the nonlinear velocity and vorticity fields, compared to the linear proper orthogonal decomposition (POD) method and the Koopman-constrained-CNN-AE using the same number or dimension of modes. We perform an analysis of the modal energy distribution of NNMs and find that compared to POD modes, the few fundamental NNMs capture a very high level of total energy of the flow, which is advantageous for reduced-order modeling and representation of the complex flows.

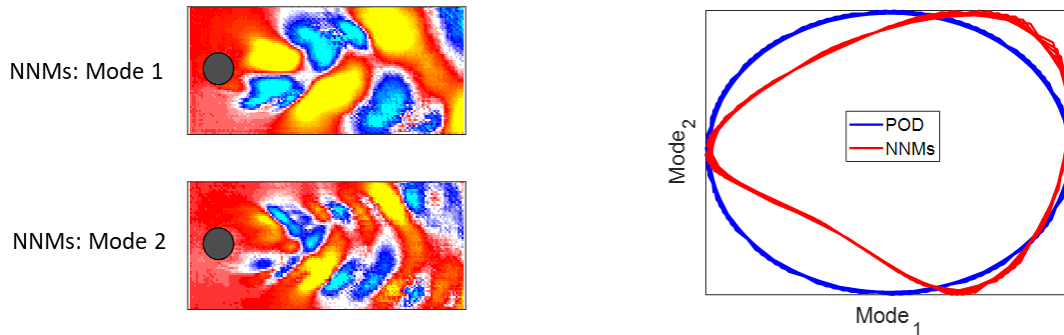


Figure 1: NNMs modes of streamwise velocity field over a cylinder and corresponding phase portraits of NNMs and POD modes

Results and discussion

Figure. 1 illustrates the identified NNMs spatial modes of a flow over a cylinder in laminar regime (streamwise velocity) and the phase portrait of NNMs modal coordinates and corresponding POD coordinates. The identified NNMs have distorted spatial patterns, capturing the nonlinear nature of the flow. This is also can be seen in the phase portrait of the trajectory of the first and second modes of POD and NNMs, where the NNMs curve is more distorted (nonlinear) while the trajectory plot of POD modes is circular. These results suggest the identified NNMs are able to reveal the nonlinear physics behind the flow in the laminar regime better compared to the linear method POD. These nonlinear spatiotemporal features captured by NNMs are also beneficial to the reduced-order reconstruction and prediction of the flow field potentially containing nonlinearity.

References

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