Approximate analytical and numerical solutions to jerk equations

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Abstract. Approximate solutions to the periodic response of non-singular, polynomial jerk equations are obtained by taking moments of the governing equations and compared with those obtained by means of the first-order harmonic balance and Galerkin procedures, and Linstedt-Poincaré techniques. The formulation makes use of analytical quadrature and integration by parts, and requires fewer conditions on differentiability than the harmonic balance procedure which is also known to require an a priori great knowledge of the solution, and does not require the presence of small parameters. The approximate solutions have been found to be in good agreement with numerical ones for a variety of thermos-mechanical oscillator models.

Introduction

Initial-value problems of third-order ordinary differential equations, also known as jerk equations, arise in the modelling of, for example, thermo-mechanical oscillators in fluids, transverse motions of piano strings, interactions between an elastic sphere and a surrounding fluid, vibrations of a mass attached to two horizontal strings and subject to aerodynamic forces, control systems, etc. [1].

Previous approaches that have applied to obtain the periodic solution of unforced, nonlinear jerk equations include the harmonic balance procedure [2] and Linstedt-Poincaré methods [3]; the latter require that a small parameter be present in the equations, whereas the former usually requires a good knowledge of the solution and is most often limited to a first-order approximation. By way of contrast, the method presented here is applicable to jerk equations of the polynomial type, is based on taking four moments of the governing equations and integration by parts, provides the frequency of the periodic motion as a function of its amplitude, may be used to determine whether an autonomous jerk equation has or has not periodic solutions, and does not require the presence of a small parameter.

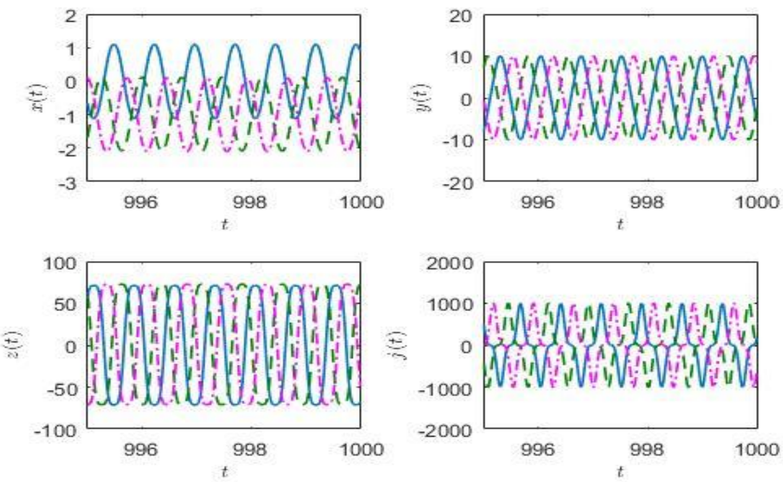

Figure 1: Displacement (x), velocity (y), acceleration (z) and jerk (j) of an oscillator immersed in a fluid as functions of time (t).

Results and discussion

Some sample results illustrating the displacement, velocity, acceleration of a thermos-mechanical oscillator immersed in a viscous fluid are shown in Figure 1 for three different initial conditions. The figure shows that the acceleration is flattened near its relative maxima and minima, while the jerk is characterized by time intervals where its value is close to zero and adjacent intervals where its slope is very large. The figure also shows that the velocity, acceleration and jerk are periodic functions whose amplitudes are almost linear, quadratic and cubic functions of the displacement frequency, respectively. The results shown in Figure 1 have been found in good accord with the numerical ones obtained by means of a fourth-order accurate, explicit Runge-Kutta method. For jerk equations which are of the polynomial type but not rational functions of polynomials, the results of the moments method presented here have been found to agree with those obtained with the harmonic balance technique, but this is not the case if the jerk equation contains ratios of polynomials.

References

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