

# Unsteady two-temperature heat transport in mass-in-mass chains

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**Abstract.** We investigate the unsteady heat transport in an infinite mass-in-mass chain with a given initial temperature profile. The chain consists of two sublattices: the  $\beta$ -Fermi-Pasta-Ulam-Tsingou (FPUT) chain and oscillators (of a different mass) connected to each FPUT particle. Initial conditions are such that initial kinetic temperatures of the FPUT particles and the oscillators are equal. The harmonic theory predicts that during the heat transfer the temperatures of sublattices are significantly different, while initially and finally (at large times) they are equal. This may look like an artifact of the harmonic approximation, but we show that it is not the case. Two distinct temperatures are also observed in the anharmonic case, even when the heat transport regime is no longer quasi-ballistic. We show that the value of the nonlinearity coefficient required to equalize the temperatures strongly depends on the particle mass ratio. If the oscillators are much lighter than the FPUT particles, a fairly strong nonlinearity is required for the equalization.

## Introduction

Far from thermal equilibrium, the concept of temperature as a single scalar parameter, characterizing local thermal state of a system, may be insufficient. Introducing several temperatures for each subsystem is often necessary. For example, in systems subjected to fast laser excitation, the temperatures of the lattice and electronic subsystem are different (see e.g. [1]). In heat conducting lattices, different temperatures can be observed in a nonequilibrium stationary state. For instance, in a one-dimensional diatomic chain limited by thermostats, the kinetic temperatures of sublattices can differ both in the cases of harmonic [2] and anharmonic interactions [3]. Evolution of kinetic temperatures during non-stationary heat transfer remains an open problem, and the current article is devoted to it. We consider an infinite chain of particles of mass interacting with the Fermi-Pasta-Ulam-Tsingou (FPUT) potential. A Duffing oscillator is attached to each particle with a mass (1).

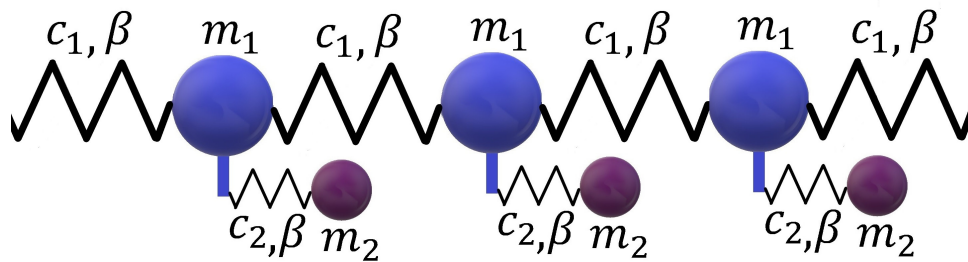


Figure 1: The mass-in-mass chain, consisting of the  $\beta$ -FPUT chain and additional nonlinear oscillators.

In the literature, such a model is referred to as a mass-in-mass chain and is extensively studied as a model of an acoustic metamaterial (see e.g. [4]). Also, the mass-in-mass can be considered as a model of chain of hydrocarbons or a system of phonons interacting with electrons. Results of the investigation may serve for the development of multi-component continuum models with several temperatures, behavior of these can be governed by a coupled system of heat transfer equations. If necessary, the reader can see [5] for details of the study.

## Results and discussion

Results of the investigation are next:

1. Closed-form formula for description of evolution of sinusoidal temperature profile in process of ballistic heat transport.
2. Characteristic features of ballistic heat transport in the weakly anharmonic chain are demonstrated.
3. Two different temperatures remain even in the case of strong anharmonicity, causing change of heat transport regimes. Maximum difference of the temperatures significantly depends on the ratio of masses.

## References

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