Detection of pull-in and periodic solutions of magMEMS model using Sturm's theorem

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Abstract. An algorithm based on Sturm's theorem is developed to predict and identify periodic and pull-in solutions for a model of magnetic actuator arising from the design of a magnetic Micro-Electro-Mechanical Structure (magMEMS) with current carrying filaments. Numerical simulations are performed to verify the proposed algorithm. The obtained results are useful for understanding the operation of single-degree-of-freedom magMEMS.

Introduction

The operation of magnetic Micro-Electro-Mechanical Structures (MEMS) is affected by a pull-in effect which occurs at certain thresholds. Detection of pull-in in the magnetostatic actuation is necessary for the design and operation of magMEMS devices. Dynamic pull-in phenomena in actuators are effects of certain combination of kinetic and potential energies resulting in the collapse of the moving structure. MEMS with magnetic actuation offer better performance than traditional MEMS with electrostatic actuation due to the generation of much larger force and fields that diminish more slowly with distance. The goal of this work is to present an algorithm that for given system parameters can predict with high accuracy the periodic solution of magMEMS model with current-carrying filaments, see Fig. 1, to avoid the dynamic pull-in.

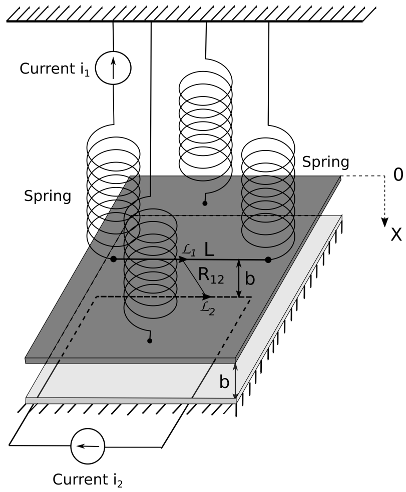

Figure 1: MagMEMS with current-carrying filaments.

Application of Sturm's theorem to detect dynamic pull-in

The motion of the wire can be described as a motion of point mass using a dynamic lumped parameter differential equation and can be derived from the second Newton's law of motion. The resulting zero initial value problem in the dimensionless form reads as follows

$$\ddot{x} + x - K\left[\frac{\xi^2\,(1-x)}{\sqrt{\xi^2\,(1-x)^2+1}} + \frac{1}{(1-x)\sqrt{\xi^2\,(1-x)^2+1}} - \xi\right] = 0, \tag{1}$$

where the actuation and geometry parameters are given by $K = \frac{\mu_0 i_1 i_2 L}{2\pi\, k\, b^2}$, and $\xi = \frac{b}{L}$, respectively, cf. [2] . It is assumeed that the currents in both wires are unidirectional, i.e., $K \geq 0$. Notice that for $\xi \to 0^+$ the motion of the filament is caused by the magnetic field of an infinite current-carrying conductor. In that case solutions are periodic if $K < K_0^*$ whereas the touch-down occurs if $K > K_0^*$ where the dynamic pull-in threshold is given by $K_0^* = 0.203632188\ldots$, cf. [1]. The solution $x(t)$ to zero initial value problem for Eq. (1) is periodic if and only if its phase diagram, x vs. $\dot{x}$, is a closed curve. In the case of small values of the parameter ξ, the existence of periodic solutions is ensured if $\tilde{f}_{K,\xi}(s) = -s^2 - 2K\log|1-s| - 2Ks\xi - \frac{1}{2}Ks(s-2)\xi^2$ has a root in $(0,1)$. The existence of roots of the function $\tilde{f}_{K,\xi}(s)$ can be precisely deduced from the Sturm's theorem.

References

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