Quasi-periodicity of temporarily constrained variable-length elastic pendulum

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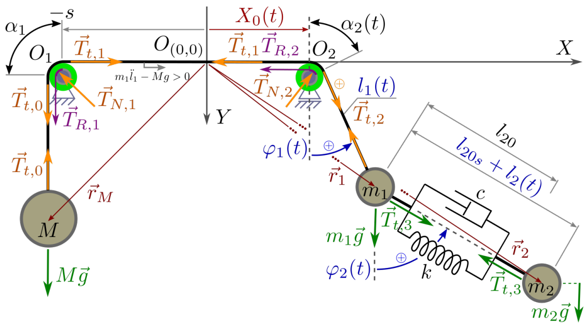
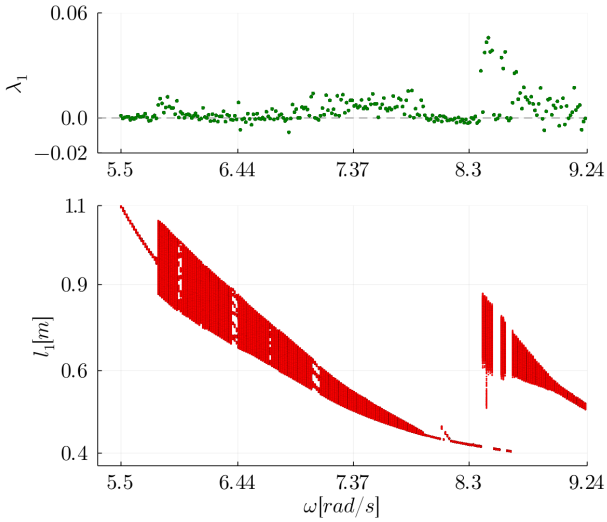
Abstract. In order to verify the very complex dynamics of temporarily constrained mechanical vibrations, a model of an elastic double pendulum of variable length with friction and a counterweight mass is introduced. The system has no physical restrictions in the form of stationary motion stops or even springs or dampers at the boundaries, constituting a dynamically unstable 4-DoF system with kinematic excitation. A series of numerical experiments based on the observation of time trajectories, bifurcation diagrams, Lyapunov exponents and the Kaplan-Yorke dimension have been computed. The bifurcation diagrams and parametric plots of the first Lyapunov exponent show that for some region of projection of Poincaré maps on a selected dimension, the system oscillates chaotically and mainly quasi-periodically in a very wide range of the control parameter of the bifurcation diagram.

Introduction

Mathematical model of the dynamical system shown in Fig. 1 is derived and simulated, where all forces and dimensions are indicated with respect to the origin O. It has two mechanisms of energy absorption: in the sliding friction with a temporary constraints at stick phases of the pulleys (green rings) and in the damper c.



(a) A physical model of the temporarily constrained dynamical system. The rotating two-pulley suspension of the system includes a parallel spring-damper coupling between the two bodies m_1 and m_2. Point O_1 is fixed, while O_2 oscillates harmonically on the line (O, X); the lengths $l_1(t)$ and $l_2(t)$ and the angles of rotation $\varphi_1(t)$ and $\varphi_2(t)$ are elements of the state vector.



(b) Bifurcation diagram $l_1(\omega)$ and the first λ-exponent.

Figure 1: Free-body diagram of the 4-DoF mechanical system (a); selected results of dynamical analysis (b)

Results and discussion

After a review of the potential fields of applications conducted in [1], a double spring pendulum has been added on the opposite side of the counter mass M. The dynamic response of this part together with the response of the whole system, consisting of three bodies, i.e. the counterweight mass M and the masses of the right-hand side pendulums m_1 and m_2, connected by a non-stretchable string suspended on pulleys, constitute the existence of dynamic instability. As a result, after conducting a dynamic analysis in a wide spectrum of changes in one of the parameters of the kinematic excitation function (see Fig. 1a), as well as observing Poincaré maps and time histories in a very long interval of time (up to duration of 10000 periods of the sinusoidal kinematic excitation), it can be concluded that the related dynamics of the motion of these bodies is mostly quasi-periodic (see Fig. 1b) On the basis of the observed bifurcations of solutions, the dynamical system under study with engineering applications states a generator of quasi-periodic solutions represented sets of points of Poincaré maps creating closed curves.

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References

[1] G. Yakubu, P. Olejnik, and J. Awrejcewicz. On the modeling and simulation of variable-length pendulum systems: A review. *Archives of Computational Methods in Engineering*, 29(4):2397–2415, 2022.