Nonlinear Reciprocal Dynamics in Systems with Broken Mirror Symmetry

Behrooz Yousefzadeh* and Andrus Giraldo**

*Department of Mechanical, Industrial & Aerospace Engineering, Concordia University, Montreal, QC, Canada

**School of Computational Sciences, Korea Institute for Advanced Study, Seoul, Korea

Abstract. Nonreciprocal devices can provide new functionalities for steering of elastic waves. Breaking the mirror symmetry is a necessary condition for enabling nonreciprocal dynamics in nonlinear devices. But the response of a nonlinear system with broken mirror symmetry is not necessarily nonreciprocal, even near the system resonances. Here, we present methodology for obtaining stable nonlinear reciprocal dynamics in coupled systems with broken mirror symmetry.

Introduction

Reciprocity relations describe invariance properties of elastic wave propagation in materials [1]. In linear, time-invariant materials, reciprocity means that the response of a material to an external load remains invariant with respect to interchanging the locations of the source and receiver. Avoiding this invariance property has posed a challenge in the design of modern devices for wave engineering. Breaking the mirror symmetry is a necessary requirement for nonreciprocal dynamics to exist in nonlinear systems. Such asymmetry is a necessary condition for enabling nonreciprocal dynamics in nonlinear systems, but it is not a sufficient condition.

In this work, we present a methodology for restoring dynamic reciprocity in coupled systems with broken mirror symmetry. We use the archetypal framework of two coupled oscillators for this purpose. The governing equations that we consider are

$$\begin{aligned} \ddot{x}_1 + 2\zeta\dot{x}_1 + x_1 + k_c(x_1 - x_2) + k_N x_1^3 &= F_1\cos(\omega_f t), \\ \mu\ddot{x}_2 + 2\zeta\dot{x}_2 + r x_2 + k_c(x_2 - x_1) + k_N x_2^3 &= F_2\cos(\omega_f t) \end{aligned} \tag{1}$$

We develop numerical continuation codes through the software AUTO to compute the steady-state response of Eq. (1) as a family of periodic orbits; see [2] for the further details. We present results corresponding to $\zeta = 0.05$, $k_c = 5$ and $k_N = 1$. Parameters μ and r control the mirror symmetry of the system.

Results and discussion

To test for reciprocity, we need to compare two configurations: (i) *forward*, where $F_1 = P$, $F_2 = 0$ and the monitored output displacement is $x_2^F(t)$, (ii) *backward*, where $F_1 = 0$, $F_2 = P$ and the monitored output displacement is $x_1^B(t)$. Reciprocity holds if and only if $x_2^F(t) = x_1^B(t)$.

Fig. 1(a) shows a special case of nonreciprocal dynamics for $\mu = 1.5$ near the second mode of the system, where nonreciprocity manifests itself as a difference in phase, but not amplitude, of the forward and backward configurations [2]. This state is characterized by a nonreciprocal phase shift, $\Delta\phi$. To achieve reciprocity, we need to simultaneously keep the amplitudes of $x_2^F(t)$ and $x_1^B(t)$ equal and set $\Delta\phi = 0$. This is only possible by using a second symmetry-breaking parameter, r. Fig. 1(b) shows the evolution of the reciprocity norm, $R = (1/T)\int_0^T (x_2^F - x_1^B)dt$, and nonreciprocal phase shift, $\Delta\phi$, as a function of r. Reciprocity is restored, $R = 0 \Leftrightarrow x_2^F(t) = x_1^B(t)$, because the two symmetry-breaking parameters are effectively balancing each other.

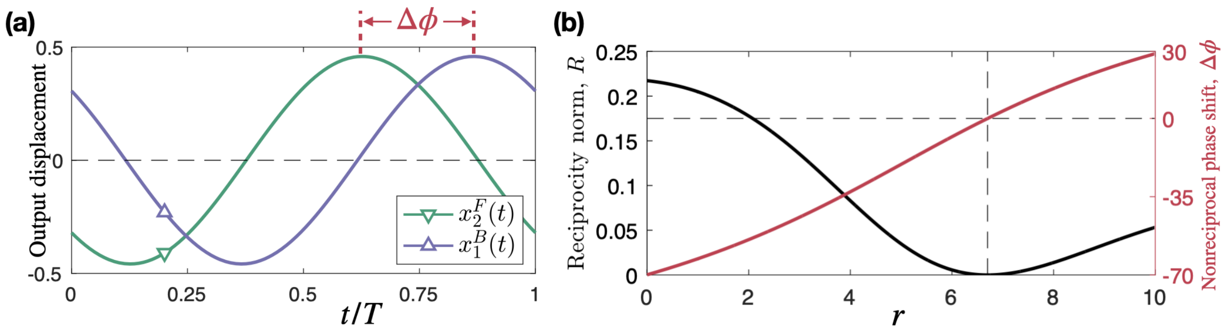

Figure 1: **(a)** Phase nonreciprocity at $P = 0.5$ with $\mu = 1.5$ as the symmetry-breaking parameter. This is achieved near $\omega_f \approx 3$. **(b)** A second symmetry-breaking parameter, r, brings nonrecipcal phase shift to zero near $r \approx 6.7$.

In conclusion, we demonstrate the possibility of restoring dynamic reciprocity in coupled nonlinear systems with broken mirror symmetry. This is achieved by simultaneously tuning two symmetry-breaking parameters such that the second symmetry-breaking parameter can counteract the original asymmetry and restore the reciprocity invariance in the system. We interpreted the results in the context of phase nonreciprocity.

References

[1] Nassar H., et al. (2020) Nonreciprocity in acoustic and elastic materials. *Nat. Rev. Mater.* **5**(9):667-685.

[2] Yousefzadeh B. (2022) Computation of nonreciprocal dynamics in nonlinear materials. *J. Comp. Dyn.* **9**(3):451-464.