Fingered Stability Regions for Operator Splitting

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Abstract. We compare the stability preserving properties of the Lie-Trotter, Strang-Marchuk, and symmetrically weighted sequential splitting by evaluating the trace and determinant of the split systems in terms the trace and determinant of the continuous system.

Introduction

Splitting methods are based on the decomposition of the underlying operator/matrix equation into a sum of simpler operators/matrices and solving a chain of these simpler problems. This method is used in various fields e.g. in fluid dynamics, for stiff systems which occur in combustion, air pollution, and reactive flow problems etc. A good exposition of splitting methods can be found in [1, 2].

The splitting literature is almost entirely dedicated to the investigation of convergence and its order of different splitting types. Our plan is different, we investigate the stability preserving properties.

Results

Consider the initial value problem

$$\dot{u}(t) = Au(t)\,,$$
$$u(0) = u_0\,,$$

with the decomposition

$$B + C = A.$$

This results in the iteration

$$\begin{cases} u_{n+1} = Su_n\,, \\ u_0 = u(0)\,, \end{cases}$$

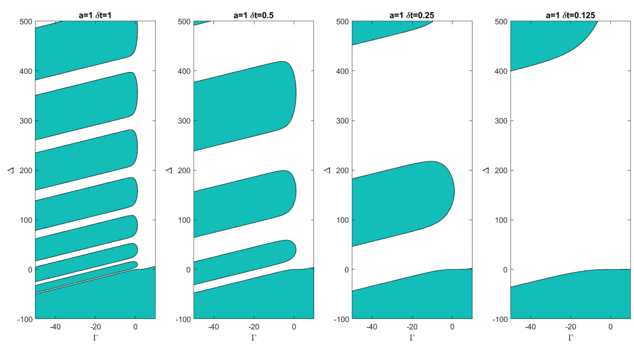
where for a fixed time-step δt the Lie-Trotter, Strang–Marchuk, and the symmetrically weighted sequential splitting methods result in the operators

$$S_{LT} = e^{B\delta t}e^{C\delta t}\,, \qquad S_{SM} = e^{\frac{1}{2}B\delta t}e^{C\delta t}e^{\frac{1}{2}B\delta t}\,, \qquad S_W = \frac{1}{2}\left(e^{B\delta t}e^{C\delta t} + e^{C\delta t}e^{B\delta t}\right)\,.$$

We worked with the simplest setting - since the literature is almost absent about the stability preserving properties of the splitting methods - when the operator A is a matrix. We consider the "natural" decomposition

$$\mathbf{A} = \begin{pmatrix} a & b \\ c & d \end{pmatrix} = \mathbf{B} + \mathbf{C} = \begin{pmatrix} a & 0 \\ 0 & d \end{pmatrix} + \begin{pmatrix} 0 & b \\ c & 0 \end{pmatrix}\,.$$

In [3] we obtained that the stability regions of the split systems exhibit fingers (see Figure), i.e. the stability is not a monotonic property of the splitting timestep δt. We characterized the thickness of the stability fingers as well as the gap between them. Both the thickness and the size of gaps grow with decreasing splitting time step.



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References

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