Chaos Measure Dynamics and a Multifactor Model for Financial Markets

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Abstract. This paper applies rolling windows to generate time-varying data series of selected chaos measures (i.e. Hurst exponent, maximum Lyapunov exponent, Lyapunov sum and sample entropy). The series are analysed to elaborate on time-varying underlying data generating process (DGP) characteristics and dynamic chaos (in)stability of the original data. Furthermore, the denoted chaos measure series are combined into a multifactor model to propose an explicative rationale for the original data set's inherent factor composition. The rolling windows are applied to cascadic (level 12) Haar-wavelet filtered daily S&P500 logarithmic returns (2000-2020), which have been shown to consist of a mixture between (hyperchaotic) deterministic and stochastic chaos. The chaos measure series are analysed by a nonlinear analysis framework, which allows the extraction of the underlying characteristics of the empirical DGP of nonlinear time-series.

Introduction

This paper represents the finale of a larger research study, encompassing five publications. Initially, a mathematical literature review with citation network analysis about financial and risk modelling has been conducted (see [1]). It analyses over 800 mathematical models, states the non-existence of a "single-best" approach and proposes nonlinear models to perform better. Thereinafter, another literature review (~160k papers) about nonlinear dynamics and financial chaos has been deduced to elaborate on the reasons for nonlinear models to outperform (see [2]). During the course of the analysis, a 40-year-old debate in nonlinear dynamics and financial chaos has been rendered visible, namely, whether the empirical underlying DGP of a given time-series follows stochastic or chaotic dynamics and how these dynamics are safely quantifiable and distinguishable. Following these insights, a novel framework built upon the given literature has been created, allowing the save quantification of the empirical DGP of any nonlinear time-series (see [3,4]), simultaneously showing the denoised daily S&P500 logarithmic return series (2000-2020) to be a mixture of (hyper)chaotic versus stochastic dynamics, which drastically diminishes forecasting potentials (see [3]). Building on these insights, the bridge between these hyperchaotic dynamics, multifractals, momentum trading, efficient markets and scaling laws has been elucidated via rolling windows and time-varying Hurst exponents (see [4]). To provide more background, a dissipative chaotic system will deflate onto its own (strange) attractor, which, if intersected with Poincaré sections, results in fractal sets. Those fractal sets are described via scaling laws. Following Berghorn [5] states that financial data follows (multi)fractal scaling laws, namely, trend inducing mechanics, which in fact cause the momentum effect. Subsequently, due to the application of rolling windows to determine time-varying Hurst exponents, the total invalidity of the efficient market hypothesis and an explication for momentum crashes, namely, the vanishing of said (multi)fractal trends during crises periods, is shown [4]. Now, if the (rolling window) Hurst exponents interpreted as a fractal trending measure are valuable and are able to show structural instabilities of a chaotic system, other chaos measures, may potentially be also yielding explicative powers. Thus, the time-varying series via rolling windows of other chaos measures such as Lyapunov exponents are derived and analysed within this study.

Results and Discussion

The time-varying chaos measures under analysis each reveal complex underlying (non-chaotic) dynamics of their own, differing from the underlying original data. Following the nonlinear framework, each series can be quantified and the empirical DGPs properly specified. Moreover, dynamical breaks or shifts (i.e. chaos instabilities) between conservative and dissipative system characteristics during (financial) crises periods can be stated. A combination of these chaos measure series in a multifactor model yields notable explicable power in terms of the composition of the S&P500 return series stating a novel way of elucidating the underlying functioning of financial markets data and potential crisis predictability.

References

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