Mean-reverting schemes for solving the CIR model

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Abstract. A family of methods for the numerical solution of the CIR model reproducing the mean-reversion property of the exact solution is presented. The convergence of the methods in the strong and weak senses is established. In addition, a method that captures exactly the first and second long-term moments of the CIR process is found.

Introduction

The Cox-Ingersoll-Ross (CIR) model describes the interest rate as the solution to the nonlinear equation

$$dX(t) = \alpha(\theta - X(t))dt + \sigma\sqrt{X(t)}dW_t \tag{1}$$

where W_t is a standard Wiener process and $\alpha, \theta, \sigma \in \mathbb{R}^+$. The solution is mean reverting: In fact, the long term first and second moments of the CIR process are given by:

$$\lim_{t\to\infty} \mathbb{E}[X(t)] = \theta, \quad \lim_{t\to\infty} \mathrm{Var}(X(t)) = \frac{\sigma^2\theta}{2\alpha}.$$

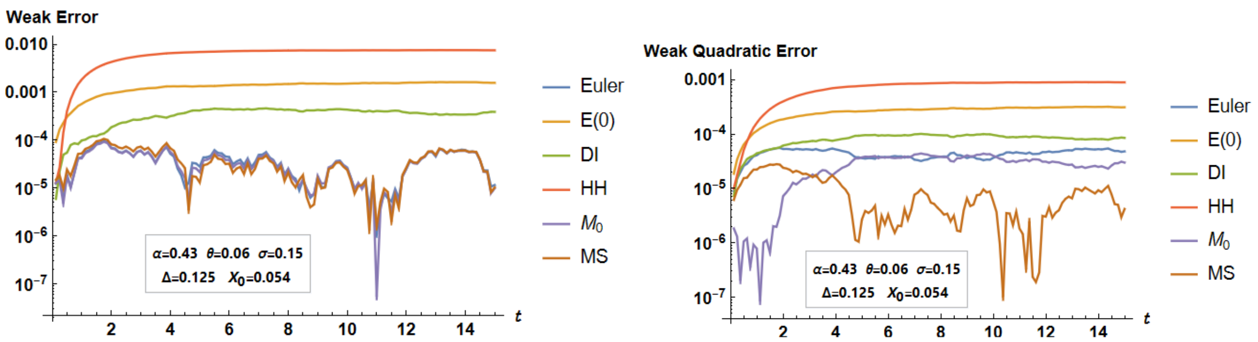
Numerical methods specially designed to solve the CIR equation have been proposed in the literature [1, 2, 3, 4]. A desirable property of any numerical method for solving an SDE is the preservation of qualitative properties of the exact solution [5]. In this sense, our goal is to propose schemes that applied to the CIR problem give numerical solutions that preserve the first and the second long-term moments.

Results and discussion

To solve numerically the equation (1), as a modification of a method presented in [1], we propose the schemes

$$X_{n+1} := \left(\left(1 - \frac{\alpha}{2}\Delta + K_\Delta\Delta^2\right)\sqrt{X_n} + \frac{\sigma}{2}\Delta W_n + S_\Delta\,\Delta\,\Delta W_n\right)^2 + \left(\alpha\theta - \frac{\sigma^2}{4}\right)\Delta$$

where $K_\Delta = \mathcal{O}\left(\Delta^0\right)$, $S_\Delta = \mathcal{O}\left(\Delta^0\right)$. We prove that these schemes converge to the exact solution in the strong and weak senses. Later we give sufficient conditions to obtain numerical solutions that inherit the mean-reverting property. Numerical experiments confirm our findings, as can be seen in the following figure where our proposed methods, M_0 and MS, preserve the first and the first two moments respectively.



References

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