Broadening the operational range of a fractionally damped piezoelectric energy harvester

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Abstract. Bimorph cantilever beams have been widely used for piezoelectric energy harvesting. However, these harvesters are efficient only in very narrow frequency ranges near resonant states. In this contribution a solution is proposed by connecting multiple beams by fractionally viscoelastic coupling layers and adding concentrated masses. The results show that the harvester operational range can be significantly broadened by the presented design procedure.

Introduction

Bimorph cantilever beams have been widely used for energy harvesting [1]. However, their operational frequency range is very narrow, since they are effective only near the resonant state. Electric power output can be improved by connecting beams at their free ends [2], but the operational range is still left narrow. Here, a solution is proposed where N_b bimorph cantilever beams are connected *along their entire length* by coupling layers as shown in Fig.1a, using fractional Kelvin-Voigt viscoelastic model with derivative order β, compliance coefficient κ and retardation time τ. When individual beams' natural frequencies are close but different, the so obtained system enters a near-resonant state more often, effectively broadening the operational range.

Results and discussion

All beams have the same geometry (cross-sectional moment of inertia I and width b, length L and piezo-layer thickness h_p) and material characteristics (mass per unit length m, elasticity modulus Y, piezoelectric constant e_{31} and permittivity ϵ_{33}), while the $k-$th beam desired dynamic properties are achieved by attaching N_{m_k} concentrated masses $m_{p_{(k)}}$ at positions x_{p_k}, where $k = 1, 2 \dots N_b$, $p = 1, 2 \dots N_{m_k}$. All piezo-layers are connected in parallel to an electric circuit of resistance R. The system is subjected to transverse base motion $w_b(t)$, inducing transverse beam displacements $w_k(x,t)$ and voltage $\mathrm{v}(t)$. For elastic Euler-Bernoulli beams, equations of motion, electric circuit and boundary conditions (BCs) are:

$$\left(m + \sum_{p=1}^{N_{m_k}} m_p \delta(x - x_{p_k})\right) \ddot{w}_k + YI\dot{w}_k'''' - \kappa\left(1 + \tau^\beta D^\beta\right)\left(w_{k-1} - 2w_k + _{k+1}\right) + e_{31} b\, h_{pc}\, \mathrm{v} = 0,$$

$$N_b \frac{\epsilon_{33} b L}{h_p} \dot{\mathrm{v}} + \frac{1}{2R}\mathrm{v} + e_{31} b\, h_{pc} \sum_{k=1}^{N_b} \int_0^L \dot{w}_k''\, dx = 0, \;\; w_k(0,t) = w_b(t), \; w_k''(L,t) = YIw_k''(L,t) = YIw_k'''(L,t) = 0$$

where D^β is the left Riemann-Liouville fractional derivative and δ is the Dirac δ function. After homogenization of the BCs and applying Galerkin discretisation, fractional derivative is approximated as in [3] and frequency domain solutions for beam displacements and voltage are determined. Fig.1b shows the frequency response function for electric voltage for the case of $N_b = 5$ beams connected by layers with $\tau = 0.2$ and various fractional derivative orders. The results show that by increasing the β, although the maximum voltage amplitude slightly drops, the system operational range broadens significantly, making the harvester more efficient for a wider range of excitation frequencies and proving the effectiveness of the presented procedure.

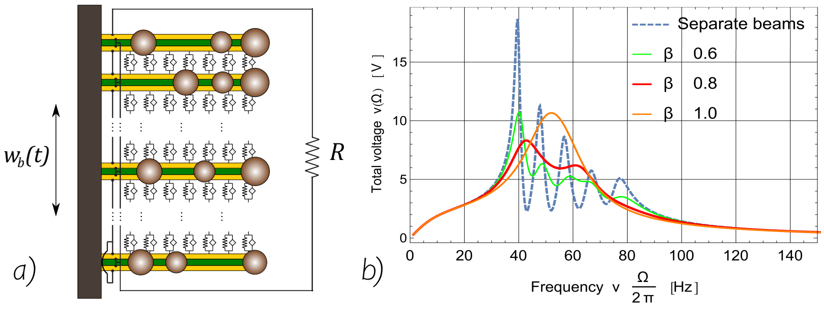

Figure 1: a) The considered system of connected bimorph cantilever beams with attached concentrated masses, b) The electric voltage frequency response function for $N_b = 5$ connected beams and various fractional derivative order for the coupling layer damping

References

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