A vorticity wave packet breaking within a rapidly rotating vortex

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Abstract. This study considers the interaction between a free vorticity wave packet and a rapidly rotating vortex in the slowly-evolving regime, a long time after the initial, unsteady, and strong interaction. The interaction starts when the amplitude modulated neutral mode enters resonance with the vortex on a spiraling critical surface, where the phase angular speed is equal to the rotation frequency. The singularity in the modal equation on this asymmetric surface strongly modifies the flow in its neighborhood, the three-dimensional (3D) helical critical layer, the region where the wave/vortex interaction occurs. Through matched asymptotic expansions, we find an analytical solution of the leading-order motion equations inside the 3D critical layer (CL). The system of the coupled evolution equations of the wave amplitude and the low-order CL-induced mean flow on the critical radius has been derived in the quasi-steady regime. The main outcome is that the wave packet/vortex interaction leads to a fast vorticity wave breaking.

Introduction

Though the motion in intense atmospheric vortices such as tropical cyclones can be considered highly axisymmetric above the surface boundary layer, observation often shows asymmetric features. The latter are generated, for instance, by the environmental wind shear, the beta effect, turbulent stress at the sea surface, and moist convection. Through a radiating wave-induced axisymmetric adjustment, these asymmetries are believed to play a significant role in the intensification of these vortices [1]. Latent heat release, for example, creates asymmetric potential vorticity (PV) anomalies that outward propagate in the form of PV waves; their breaking was recently related to the inner spiral rainbands. Observation and numerical simulations indeed show that inner spiral bands mainly exhibit vorticity wave characteristics [2].

Understanding the dynamics of these spiral bands and their contribution to the vortex evolution can greatly help improving the prediction of violently rotating vortex intensity. Wave activity analysis in numerical hurricane-like vortex models shows that vortices only interact with vorticity waves and that the related modes are continuous, that is, admit a CL singularity. This study therefore wishes to improve the understanding of the nonlinear dynamics of continuous vorticity modes embedded in rapidly rotating vortices. In particular, it examines the complex dynamical coupling between a vorticity wave packet and the azimuthally averaged 3D wind through the nonlinear CL theory in the quasi-stationary state assumption. In the previous analytical studies dealing with 2D or 3D, nonlinear and singular wave packets, the induced mean flow was however omitted, which was a stringent mistake.

Results and discussion

This interaction generates a vertically sheared 3D mean flow of higher amplitude than the wave packet. The chosen envelope regime assumes the formation of a mean radial velocity of the same order as the wave packet amplitude, deviating the streamlines in a spiral way with respect to the rotational wind. The critical layer pattern, strongly deformed by the mean radial velocity, loses its symmetries with respect to the azimuthal and radial directions [3]. The knowledge of the wave amplitude, the leading-order mean axial and azimuthal velocity, and axial vorticity evolutions at the critical radius can be simply determined from three first-order differential equations. Numerical simulations of the first-order mean flow truncated system show that the wave packet and vortex kinetic energies slightly grow inside the envelope before the breaking onset in most of the cases, whereas the vortex was intensifying at the expense of the wave packet in the previous and unsteady interaction. The vertical wind shear has the highest effect on the wave/mean flow interaction. When the shear is moderate, it enhances intensification but when it is very large, it prohibits it in both the unsteady and slowly evolving stages [4]. Including the second-order mean flow in this system could, however, avoid the breaking and would permit the interaction to generate an asymptotic constant-speed travelling coherent vortical structure.

References

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