

Control via Nonlinear Feedback Linearization with Machine Learning

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Abstract. We use machine learning to implement feedback linearization for the control of nonlinear discrete-time systems. The proposed approach is based on physics-informed neural networks, thus implementing pole-placement and feedback linearization in one step. To demonstrate the efficiency of the scheme we used a nonlinear discrete time model for which the feedback linearization law can be derived analytically.

Introduction

Feedback linearization of nonlinear systems is arguably one of the most used nonlinear control techniques [1, 2]. Building in previous work [1, 2], we propose a Physics-informed machine learning (PIML) scheme that learns a feedback linearizing control law and performs pole placement in one step for discrete time systems of the form:

$$x(t+1) = f(x(t), u(t))$$

Thus we seek for a transformation S such that $z = S(x)$ is coupled with a control law $u = -cz = -cC(x)$ in order to linearize the above system as:

$$z(t+1) = Az(t),$$

where A has the proper set of eigenvalues to ensure stability [1].

Results and discussion

To illustrate the performance of the PIML, we consider the following system of discrete equations [1]:

$$\begin{aligned} x_1(t+1) &= \exp(0.3x_2(t))\sqrt{1+x_1(t)+x_2(t)} - 1 - 0.4x_2(t) + 0.5u(t) \\ x_2(t+1) &= 0.5\ln(1+x_1(t)+x_2(t)) + 0.4x_2(t) \end{aligned} \quad (1)$$

It can be shown [1] that the sought transformation reads:

$$T(x_1, x_2) = [\ln(1+x_1+x_2) \quad x_2] \quad (2)$$

Then $T_1(x_1, x_2) = \ln(1+x_1+x_2)$ in (2) is the desired feedback linearizing control law, where the closed loop poles are governed by the eigenvalues of matrix A here set to $k_1 = 0.8405$ and $k_2 = 0.0595$. Figure (1), shows the approximation of S obtained with the proposed PIML scheme. As it is shown, the scheme is able to learn the transformation in (2) with a numerical approximation accuracy of the order of 10^{-3} .

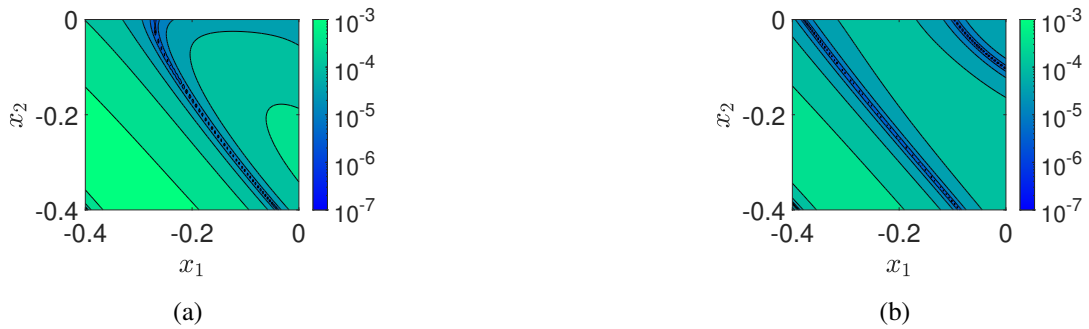


Figure 1: Numerical approximation accuracy (NAA) between the theoretical sought transformation (2) and the one learned by the PIML scheme. Panel (a) depicts the NAA for the first component $T_1(x_1, x_2)$ and panel (b) refers the same for the second component of $T(x_1, x_2)$.

References

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