Control via Nonlinear Feedback Linearization with Machine Learning

Hector Vargas Alvarez**, Gianluca Fabiani** and Nikolas Kazanztis**, Constantinos Siettos** and Ioannis G. Kevrekidis**

Scuola Superiore Meridionale, Università degli Studi di Napoli Federico II, Naples, Italy
Scuola Superiore Meridionale, Università degli Studi di Napoli Federico II, Naples, Italy
Dept. of Chemical Engineering, Worcester Polytechnic Institute, Massachusetts, USA
Dept. of Mathematics and Applications, Università degli Studi di Napoli Federico II, Naples, Italy
Dept. of Bio and Chem. engineering, Johns Hopkins University,Maryland, USA

Abstract. We use machine learning to implement feedback linearization for the control of nonlinear discrete-time systems. The proposed approach is based on physics-informed neural networks, thus implementing pole-placement and feedback linearization in one step. To demonstrate the efficiency of the scheme we used a nonlinear discrete time model for which the feedback linearization law can be derived analytically.

Introduction

Feedback linearization of nonlinear systems is arguably one of the most used nonlinear control techniques [1, 2]. Building in previous work [1, 2], we propose a Physics-informed machine learning (PIML) scheme that learns a feedback linearizing control law and performs pole placement in one step for discrete time systems of the form:

$$x(t+1) = f(x(t), u(t))$$

Thus we seek for a transformation S such that $z = S(x)$ is coupled with a control law $u = -cz = -cC(x)$ in order to linearize the above system as:

$$z(t+1) = Az(t),$$

where A has the propoer set of eigenvalues to ensire stability [1].

Results and discussion

To illustrate the performance of the PIML, we consider the following system of discrete equations [1]:

$$\begin{aligned} x_1(t+1) &= exp(0.3x_2(t))\sqrt{(}1 + x_1(t) + x_2(t)) - 1 - 0.4x_2(t) + 0.5u(t) \\ x_2(t+1) &= 0.5ln(1 + x_1(t) + x_2(t)) + 0.4x_2(t) \end{aligned} \tag{1}$$

It can be shown [1] that the sought transformation reads:

$$T(x_1, x_2) = [ln(1 + x_1 + x_2) \quad x_2] \tag{2}$$

Then $T_1(x_1, x_2) = ln(1+x_1+x_2)$ in (2) is the desired feedback linearizing control law, where the closed loop poles are governed by the eigenvalues of matrix A here set to $k_1 = 0.8405$ and $k_2 = 0.0595$. Figure (1), shows the approximation of S obtained with the proposed PIML scheme. As it is shown, the scheme is able to learn the transformation in (2) with a numerical approximation accuracy of the order of 10^{-3}.

(a) (b)

Figure 1: Numerical approximation accuracy (NAA) between the theoretical sought transformation (2) and the one learned by the PIML scheme. Panel (a) depicts the NAA for the first component $T(x_1, x_2)$ and panel (b) refers the same for the second component of $T(x_1, x_2)$.

References

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