# Fractal Response of a Nonlinear Packaging System

**Luis M. Palacios-Pineda***, Óscar Martínez-Romero**, Daniel Olvera-Trejo**, and Alex Elías-Zúñiga**

** Tecnológico Nacional de México / Instituto Tecnológico de Pachuca, Pachuca, HGO, MÉXICO*

*** Institute of Advanced Materials for Sustainable Manufacturing, Tecnologico de Monterrey, Monterrey, NL, MÉXICO*

**Abstract**. In this paper the impact curves, obtained from the dropping of a weight over a viscoelastic fractal cushioning packaging system are computed to evaluate the dropping damage of a critical component. To capture high-frequency harmonic components observed during the impact time span, the approximate frequency-amplitude expression of the corresponding equation of motion is obtained via an ancient Chinese algorithm and Jacobi elliptic functions.

## Introduction

The equation of motion that capture the fractal cushioning dynamic behavior of anti-vibration fractal materials can be assumed to be of the form [1]:

$$m\,\frac{d}{dS^{\alpha}}\left(\frac{dy}{dS^{\alpha}}\right)+g(y,S)=0, \tag{1}$$

with initial conditions:

$$y(0)=0, \qquad \dot{y}(0)=v_0. \tag{2}$$

Here $\alpha$ is a fractal parameter related to the cushioning surface porosity. Equation (1) provides information about the shape, duration, and peak magnitudes of the displacement, velocity, and acceleration curves that appear when a weight is dropped over at a certain height over a cushioning surface. Equation (1) can be also written as an ordinary differential equation introducing the fractal derivative definition

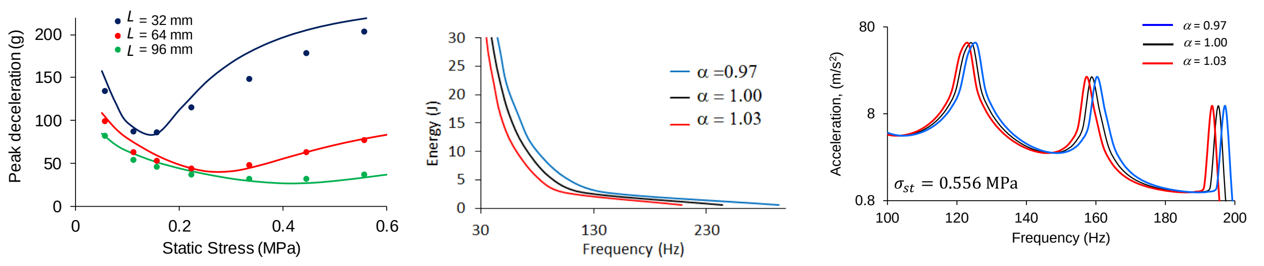
$$\frac{dy}{dS^{\alpha}}(S_0^{\alpha})=\Gamma(1+\alpha)\lim_{\substack{(S-S_0)\\ \Delta S\neq 0}}\frac{y(S)-y(S_0)}{(S-S_0)^{\alpha}} \tag{3}$$

and the two-scale dimension transform [2] $t=S^{\alpha}$ to get

$$m\,\frac{d^2y}{dt^2}+g(y,t)=0, \qquad y(0)=0, \qquad \frac{dy(0)}{dt^{1/\alpha}}=v_0 \tag{4}$$

## Results and discussion

Figure 1a shows the system's acceleration response evolution as the static stress increases. For low static stress values, the dynamic response is less complex when compared with those of higher static stress values. The fractal evolution on the cushioning material energy, as the frequency increases, is shown in Fig. 1b. Also notice from Fig. 1c that for higher cushioning material porosity values of $\alpha$, the impact traveling wavelength increases, helping to reduce potential damages in the critical component which is consistent with observed practical applications.




a) b) c)

Figure 1: a) Configurational representation of the nonlinear modal amplitudes, b) frequency-amplitude response plot, and c) frequency-energy plot (FEP) that illustrates how the energy is transferred between modes.

This work unveils the advantages of modeling the dynamics of cushion materials considering their molecular fractal structure, adopting a fractal equation of motion, and the two-scale fractal dimension transform as well as the fractal energy-frequency response curves. The fractal dimension parameter values of $\alpha$ elucidates how the energy produced during impact and the acceleration response for different values of static stresses are influenced by the surface porosity of the packaging cushioning system.

## Reference

[1] Elías-Zúñiga A. · Palacios-Pineda L. M., Martínez-Romero O., Olvera-Trejo D. (2022) Dynamic response of a fractal cushioning packaging system. *Fractals* **30** 7: 2250148.