Application of SINDy for the discovery of governing equations of a trapped particle in an acoustic radiation force field

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Abstract. The contactless technique for trapping, handling, and levitating particles using acoustic radiation forces has many applications in engineering and medicine. Recently, Brunton et al. proposed the Sparse Identification of Nonlinear Dynamics (SINDy) approach to identify governing equations of motion from time series data. We use SINDy to extract the equation of motion of a trapped particle oscillating in an acoustic radiation force field which is related to the family of Duffing oscillators. We simulate the responses of this equation to benchmark SINDy for different dynamical regimes which are then validated for the case of a predicted period-1 limit cycle using experimentally captured data for TinyLev.

Introduction

Although the acoustic radiation force as a nonlinear acoustic phenomenon has been investigated for many years, the interaction of the influence of secondary excitation on the dynamics of a particle in an acoustic force field is yet to be fully studied [1]. By periodically exciting the acoustic force field externally using a shaker with amplitude, A_{ex}, and frequency, ω_{ex}, the particle is also excited with a specific amplitude, $A = f(A_{ex})$, and frequency, $\omega_0 = g(\omega_{ex})$, when f and g being functions relating the external excitation to the particle's oscillation. We explicitly derive that the developed acoustic trap follows the behaviour of a nonlinear Duffing-like oscillator as follows:

$$\ddot{\theta} + a_1|\dot{\theta}|\dot{\theta} + a_2\theta - \frac{a_2}{6}\theta^3 = F\cos\theta\sin\omega_0 t, \tag{1}$$

where θ is the displacement, '$(\cdot)$' is its time derivative, and a_1 and a_2 are coefficients depending on the drag coefficient, the levitated object, and its surrounding fluid properties, and F is a coefficient depending on the A_{ex}. Then, the Sparse Identification of Nonlinear Dynamics (SINDy) algorithm [2,3] is employed (schematic Fig 1a) to extract a set of coefficients representing the particle's equation of motion, see Eq. (1) [4]. Real-life measurements of the oscillations of a Styrofoam bead provide experimental data for a TinyLev [5] system to validate the performance of a period-1regime using $A_{ex} \leq 0.2$ [mm] and $f_{ex} \leq 100$ [Hz].

Results and discussion

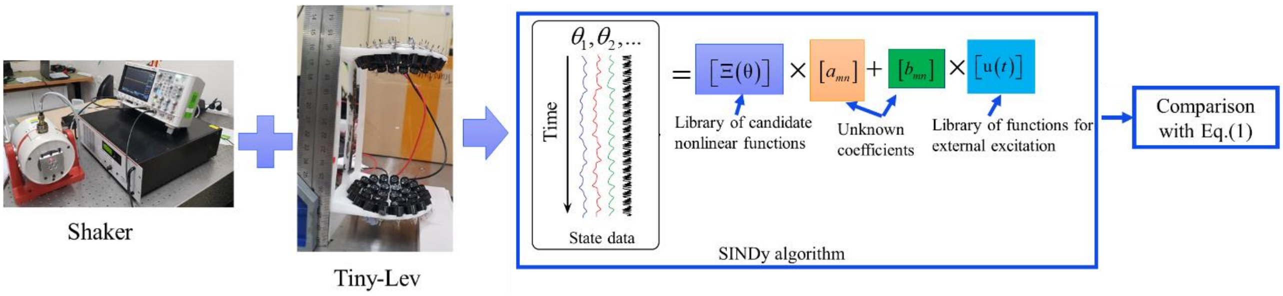

Figure 1: Schematic of SINDy algorithm to identify governing equation from nonlinear time series data. Data are collected from measurements of the system, including a time history of the states θ and derivatives $\dot{\theta}$. $[a_{mn}]$ and $[b_{mn}]$ are unknown coefficients.

It is demonstrated that SINDy is a powerful new technique to identify nonlinear dynamical systems from data without assumptions on the form of the governing equation of motion. Results indicate good correlation with theory, however moving into more complex dynamics at lower excitation amplitudes [4] indicate that higher order nonlinear terms in Eq. (1) might be required.

References

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