A Nonlinear Piezoelectric Shunt Absorber with 2:1 Internal Resonance

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Abstract. We present a theoretical and experimental analysis of a new nonlinear piezoelectric shunt absorber designed to attenuate the vibration of an elastic structure under external excitation. This absorber is formed by connecting the elastic structure via a piezoelectric patch to an electrical shunt circuit consisting of a resonant shunt combined in series with a nonlinear voltage component, that includes a quadratic and a cubic components, and with its natural frequency tuned to half that of the target mechanical mode. As a consequence, a two to one internal resonance occurs, generating a strong coupling between the mechanical mode and the electrical mode that leads to the creation of a nonlinear antiresonance that replaces the mechanical resonance of the elastic structure. The antiresonance amplitude is also subjected to a saturation, thus becoming independant of the input amplitude and giving this absorber an advantage over the standard linear absorbers. The presentation will describe the shunt circuit and its optimization, its main quadratic voltage component, and the use of the normal form theory to adjust the value of the cubic component, necessary to improve the performance of the absorber.

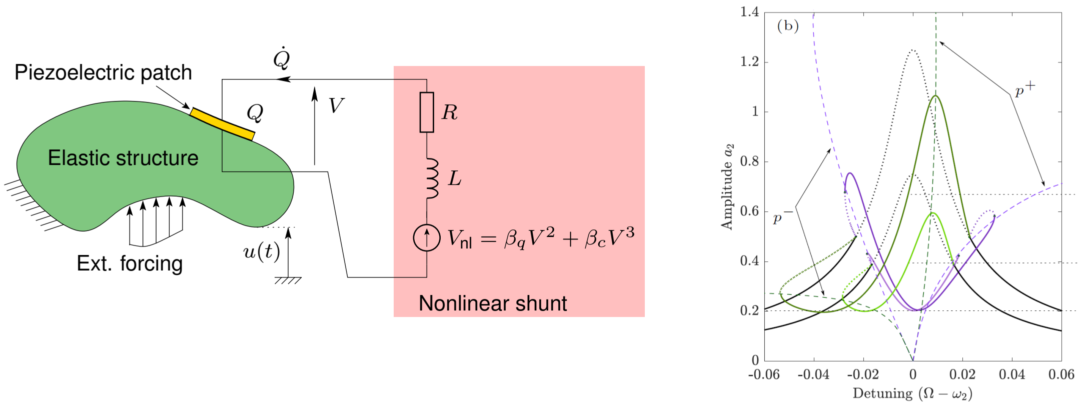

Figure 1: (left) elastic structure coupled to the shunt circuit; (right) Resonance curve of the elastic structure around the target resonance; (black curves): response without the shunt circuit; (green curves): response with the quadratic shunt; (purple curves): response with quadratic and cubic shunt.

We consider an arbitrary elastic structure subjected to an external excitation and connected to a nonlinear shunt circuit via a piezoelectric patch (PE Patch) as shown in Fig. 1. The inductance L in the shunt circuit, together with the capacitance C of the PE patch, create an electrical resonator that is tuned at half a target resonance: $\omega_e = 1/\sqrt{LC} \simeq \omega_m$, where ω_e and ω_m are the natural frequencies of the electrical resonator and the elastic structure. Then, the voltage source $V_{\text{nl}} = \beta_q V^2 + \beta_c V^3$ (with V the voltage across the terminals of the PE patch and $\beta_q, \beta_c \in \mathbb{R}$ two gains) creates quadratic and cubic nonlinear terms, in order to activate a 2:1 internal resonance between the elastic mode at ω_m and the electrical mode at ω_e. In practice, a semi-passive circuit made of operational amplifiers is used [1, 3].

A two degree of freedom model of the system in Fig. 1 can be written and solved either by numerical continuation methods and analytical perturbation methods. This enable to first show that the figure of merit of the system is the term $\xi_e/(\kappa\beta_q)$ where ξ_e is the damping ratio of the electrical circuit and κ the piezoelectric coupling factor: thus, increasing the gain β_q improves the performances and can also counterbalance, in a certain extent, a high ξ_e and a low k [2]. However, it can also be shown that a quadratic only shunt (*i.e.* with $\beta_c = 0$) creates the 2:1 internal resonance (which is the desired goal) but also creates some nonresonant terms in the system with a high value, that leads to a detuning of the shunt dependent of the amplitude. Then, properly using the normal form theory and a second order multiple scale analysis, one can compute the nonlinear mode of the coupled system (the backbone curves) and use them to show that the cubic term can be tuned, by adjusting β_c such that $\beta_c = 10\beta_q^2/9$, so as to cancel the effect of the unwanted nonresonant terms and keep the tuning and the saturation phenomenon at high levels of input [4].

References

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