Stabilization mechanism of limit cycle oscillation using control based continuation and phase locked loop.

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Abstract. This presentation concerns the stabilization of unstable limit cycles using a combination of feedback and phase locked loop control. The proposed control scheme is applied to a Van der Pol oscillator and the stabilization mechanisms are investigated theoretically using the method of multiple scales. Different failure scenarios of the controller are revealed allowing us to express design rules for the controller gains.

Introduction

Control based continuation (CBC) is used to study the bifurcation diagrams of a nonlinear dynamical systems on physical experiments [1]. CBC rely on a feedback controller that is used to stabilize the nonlinear device. The feedback controller must be non-invasive to ensure that a periodic solution of the feedback controlled system is a also a solution of the uncontrolled system. CBC has been mostly used to track branches of periodic solutions of non-autonomous (i.e. forced) systems [2]. CBC of autonomous systems presents an additional difficulty since the frequency of the limit cycle (LC) is also an unknown. Numerical continuation algorithms solve this problem by appending the system with a phase condition. In this study, the frequency of the limit cycle is determined in real time by using a phase locked loop controller (PLL). Although CBC has already been tested on various experiments, only few papers deal with the detailed analysis of the underlying stabilization mechanism.

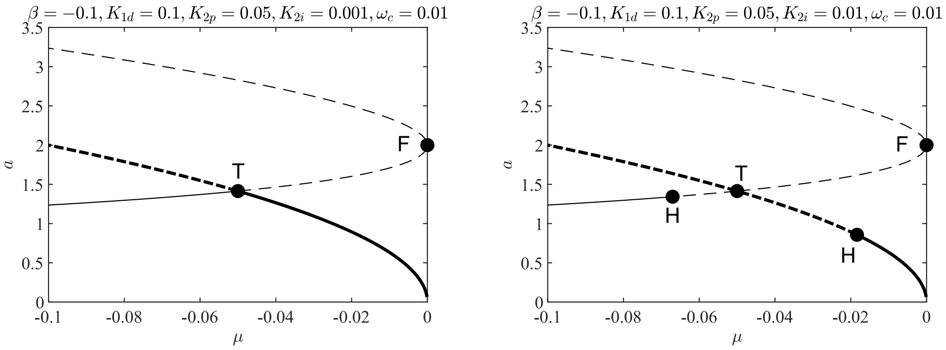

Figure 1: Bifurcation diagram of the controlled VdP

Results and discussion

The proposed control scheme has been applied to a Van der Pol oscillator (VdP). The governing equations of motion are given by

$$\begin{aligned} &\ddot{x} + x - (\mu - \beta x^2)\dot{x} = K_{1d}(\dot{w} - \dot{x}) && \dot{y}_1 = \omega_c(x\cos\theta - y_1) \\ &\dot{y}_2 = Ry_1 && \dot{\theta} = \omega_0 + K_{2p}y_1 + K_{2i}y_2 \end{aligned} \tag{1}$$

where μ and β are the linear and nonlinear coefficients of the VdP oscillator. K_{1d} is the derivative gain, ω_c the cutoff frequency of the low pass filter of the PLL, R, K_{2p}, K_{2d} are the gains of the PLL and ω_0 the center frequency. $w(t) = W\sin\theta(t)$ is the control target. The uncontrolled VdP oscillator has a Hopf bifurcation at $\mu = 0$, which is subcritical (supercritical) for $\beta < 0$ ($\beta > 0$). The behavior of the controlled VdP has been analyzed by using the method of multiple scales. Since the objective is to stabilize the unstable LC, we considered that the control target amplitude W is equal to the amplitude of the uncontrolled LC, i.e. $W = a_0 \equiv 2\sqrt{\mu/\beta}$. Examples of bifurcation diagram are depicted in fig. 1. Solid (dashed) lines correspond to stable (unstable) solutions. Thick and thin lines correspond to solutions where the controller is non-invasive ($W = a$), or invasive ($W \neq a$), respectively. Multiple scales analysis allows us to express design rules for the controller. Either the controller may fail due to the presence of a transcritical bifurcation at $\mu = -K_{1d}/2$ or due to a Hopf bifurcation if $K_{2p}(K_{1d} + 2\omega_c) < 2RK_{2i}$.

References

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