

# Embodied hydrodynamic sensing and estimation using Koopman modes

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**Abstract.** The ability to sense flows and estimate some features in the near field is very important for underwater robot. Many species of fish can accomplish such sensing even when they are blinded using their lateral line or kinematics of fins. The complexity and high (infinite) dimensionality of fluid flows around a swimmer require new methods for such flow sensing. Recent advances on Koopman operator of a dynamical system combined with machine learning offers a new way to extract useful information of the flow based on pressure or kinematic measurements from a swimmer's body. We present experimental and computational results of a trailing hydrofoil sensing and estimating the wake Strouhal number and the distance of an upstream body using on board measurements.

## Introduction

Objects moving in water or stationary objects in streams create a vortex wake. An underwater robot encountering the wake created by another body experiences disturbance forces and moments. These disturbances can be associated with the disturbance velocity field and the bodies creating them. Essentially the vortex wakes encode information about the objects and the flow conditions. Underwater robots that often function with constrained sensing capabilities can benefit from extracting this information from vortex wakes. We consider the problem of the estimation of the spatial location of an up stream obstacle or oscillating body in a flow past a pitching hydrofoil and a the reconstruction of the near body flow. It is assumed that pressure on the surface of the hydrofoil can be measured at fixed locations on the body along with the pitch angular velocity of the hydrofoil. Using time series pressure measurements on the surface of the hydrofoil and the angular velocity of the hydrofoil, a Koopman operator can be constructed that propagates the snapshots of data forward in time. The modes from a spectral decomposition of this operator then extracts important features from the measurements and can be the inputs for machine learning to estimate features of the flows and obstacles.

## Results and Discussion

Model reduction and reconstruction of reduced order models in such complex dynamical systems where only limited data on observables is available can be possible via the framework of the Koopman operator, a topic that has attracted much attention in recent years [1, 2]. We proposed a framework (see fig. 1) for sensing flows using measurements from a body immersed in the fluid in [3]. In numerical simulations of flow past an upstream body, pressure is measured on the surface of a downstream pitching hydrofoil along with its pitch angular velocity. Denoting the observables as  $g(t) = [P_1(t), \dots, P_N(t), \Omega(t)]^T$  where  $P_i(t)$  denotes the pressure on the foil at location  $i$  at time  $t$  and  $\Omega(t)$  denotes the foil angular velocity similarly at time  $t$ , the Koopman operator  $\mathcal{K}$  is a linear operator  $\mathcal{K} : L^2 \mapsto L^2$  and propagates the observables forward in time  $\mathcal{K}_T g(t) = g(t + T)$ . The eigenvalues and eigenvectors (or the singular values and singular vectors) of this operator extract features of the measurements. These feature vectors are then used as an input to deep networks to estimate the obstacle distance [3]. The results however go beyond the estimation of the distance to the obstacle as in [3], but instead show that reconstruction of the flow in subdomain containing the trailing body is possible from the Koopman modes obtained from observables from the hydrofoil. These results are a significant addition to the literature on Koopman modes and flow reconstruction since measurements are obtained from the body alone and not the fluid domain and thus have relevance to autonomous robots.

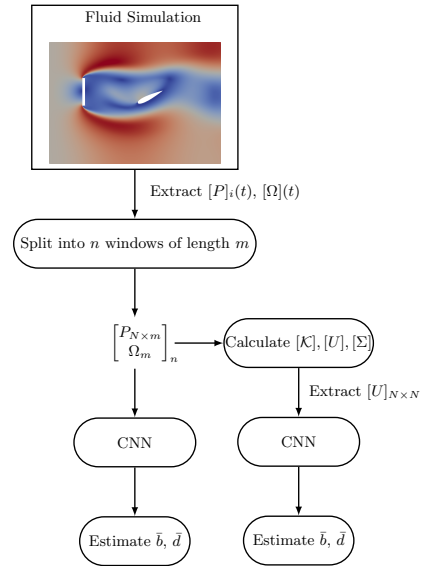


Figure 1: A Koopman operator is constructed using pressure measurements from the surface of the trailing body. The modes of this operator are used to train a deep network to estimate the obstacle distance.

## References

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