Computing Normal Forms of quadratic differential algebraic equations

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Abstract. In this paper, we present a general way of computing reduced normal form of nonlinear dynamic systems. The proposed method relies on a so-called quadratic recast in order to transform the dynamical equations into a quadratic DAE and then compute a reduced normal form of the DAE. We present an elegant (and easy to implement) method to write and solve the homological equation, that relies on linear algebra in the vector space of (finite degree) multivariate polynomials. An example is considered to illustrate the results of the method. This constitutes a general way of computing normal form that is aimed to be implemented into the MANLAB package.

Introduction

We consider computation of reduced order models of nonlinear dynamic system using the normal form theory [1, 2, 3]. Usually, the normal form method is presented by considering a system with only polynomial nonlinearity (or a Taylor expansion of the nonlinearity). To increase the generality of the normal form, we propose to consider that the initial dynamic system is under the form of a *DAE with quadratic nonlinearity only*. This is the same hypothesis, as is the MANLAB package, which allows for the application of the Asymptotic Numeric Method on a wide variety of systems [4]. Although this seem restrictive, it can be shown that most nonlinear systems can be written under this form, provided that one includes enough auxiliary variables in the so called quadratic recast [4].

Results and discussion

We consider the quadratic DAE: $A\dot{y} = Ly + Q(y,y)$, where y is the vector of unknows containing N generalized positions, N generalized velocities and M auxiliary variables (Lagrange multipliers from mechanical constraints or auxiliary variables arising from the quadratic recast). A is the mass matrix (of size $2N + M$, possibly singular), L is a linear operator (matrix of size $2N + M$) and Q a quadratic operator. For the normal form computation, we are searching for: (i): a change of coordinates $y = W(z)$ where $z \in C^n$ is a set of (complex) normal variables with n elements (n even, and usually $n \ll 2N$), and (ii) a (reduced) dynamic function $f(z) \in C^n$ for the normal variables, such that: $\dot{z} = f(z)$. Substituting the expression for the normal dynamics and the change of variable into the original DAE leads to the following homological equation: $A(\nabla_z W)f = LW(z) + Q(W(z), W(z))$. Both the change of coordinates and the reduced dynamics are considered to be (multivariate) polynomials of given degree d so that they can be written as: $W(z) = \sum W_i z^{\alpha_i}, W_i \in C^{2N+M}$ and $f(z) = \sum f_i z^{\alpha_i}, f_i \in C^n$. The computation of the normal form then reduces in finding the vectors of coefficients W_i and f_i for each monomial z^{α_i} up to degree d. This is realized by balancing the coefficients of each monomial z^{α_i} in the homological equation. At first order, the equations are associated to the linear monomials and can be solved using the linear eigen-modes of the system. At higher orders, one has to solve an equation of the form: $(A\sigma_i(f) - L)W_i + A\sum_{k=1}^{n} Y_k f_{i,r} = R_i(W)$. The idea is to have a reduced dynamic under its simplest form, so most of the coefficients of f should be zero. However, when $(A\sigma_i(f) - L)$ is singular one needs to include terms associated to resonant modes in the reduced dynamics. The strategy is sequential: the system is solved for each monomial of a given degree, and then the operation is repeated iteratively for the monomials of the next degree until the maximum degree d has been reach. Finally, several examples will be considered (Fig.1).

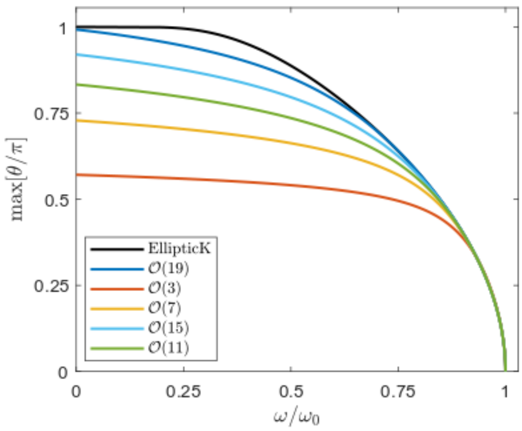

Figure 1: Backbone curve for the pendulum obtained with the normal form of the quadratic DAE at several order.

References

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