An Equality-Based Formulation for Vibrating Systems with Two-Dimensional Friction

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Abstract. A new compact and efficient formulation is presented for the construction of periodic solutions of vibrating systems with two-dimensional dry friction occurrences. Rather than relying on penalization or regularization, the approach is based on an exact formulation of the non-smooth governing equations as equalities, and periodic solutions are sought in a weighted residual sense via a Ritz-Galerkin projection. To increase efficiency, the Jacobian of the friction forces is calculated in a piecewise linear fashion. The implementation is straightforward, as it relies on basic integral quadrature schemes and existing nonlinear solvers, and does not suffer from typical limitations or hypotheses. The theory is applied to a system with two-dimensional friction contact, demonstrating its simplicity and effectiveness.

Introduction

Non-smooth nonlinearities due to unilateral contact and dry friction are ubiquitous in structural engineering systems. Turbomachinery rotors are a prime example of industrial systems that are subject to intermittent contact and feature dry friction dampers to mitigate adverse vibrations. While predicting the dynamic response of non-smooth systems is of great importance and has been the subject of much research over the years, their equations of motion can be remarkably challenging to solve, because Signorini unilateral contact and Coulomb friction conditions are inequalities. Existing methods commonly rely on the penalization of the friction force by introducing a finite stiffness or on the smoothing of the contact force [1]. Frequency-domain formulations require the calculation of the contact conditions in the time domain at each iteration of the nonlinear solver via an FFT [2], while time-domain methods mandate advanced time-stepping or event-driven schemes [3].

Results and discussion

Here the fundamental approach introduced in [4] is extended to systems with two-dimensional frictional occurrences. The key idea is to express the equations governing the non-smooth friction terms as *equalities* rather than inequalities. This enables the construction of periodic solutions in a weighted residual sense, by (i) expanding all unknowns of the problem in terms of an appropriate truncated basis of periodic functions (here the Fourier basis), (ii) projecting the equations of motion, which feature only equalities, onto the Fourier basis functions, and (iii) solving iteratively the resulting system of time-independent nonlinear equations.

The integrals generated by the projection procedure are computed numerically using a classical quadrature scheme such as a Riemann sum, and the resulting nonlinear equations are solved using a non-smooth Newton or hybrid Powell solver. A key advantage is that no penalization or regularization hypothesis is made. Also, there is no need for an FFT-based alternating frequency-time procedure to calculate the Coulomb conditions in the time domain [1] or for artificially introducing dissipation to construct the periodic solutions [2]. A typical periodic response at the friction point is depicted in Figure 1, which shows that the sticking and slipping phases of the motion are captured.

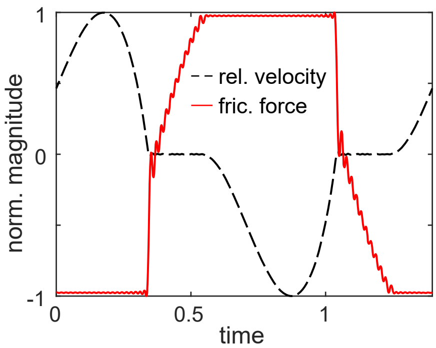

Figure 1: Periodic solution with sticking (velocity $= 0$) and slipping (force $= \pm 1$).

Two major advances are presented. First, the Jacobian is expressed in an exact, piecewise linear fashion instead of being computed numerically in the nonlinear solver, making computations significantly more efficient and accelerating convergence. Second, the formulation is applied to a two-dimensional frictional surface, as opposed to a line. Friction conditions are written as two coupled equalities (as opposed to a single equality for a contact line), which are solved for in a weak sense using the Ritz-Galerkin projection. Results show that the formulation is remarkably simpler and more powerful than existing methods. In particular, the direction of sliding motion on the contact surface becomes an unknown of the problem, alleviating the need for cumbersome hypotheses and procedures required by current methods regarding the sliding direction.

References

[1] Cameron T.M., Griffin J.H. (1989) An Alternating Frequency/Time Domain Method for Calculating the Steady-State Response of Nonlinear Dynamic Systems. *J. Appl. Mech.* **56(1)**:149-154.

[2] Nacivet S., Pierre C., Thouverez F., Jézéquel L. (2003) A Dynamic Lagrangian Frequency–Time Method for the Vibration of Dry-Friction-Damped Systems. *J. Sound Vib.* **265**:201-219.

[3] Acary V., Brogliato B. (2008) Numerical Methods for Nonsmooth Dynamical Systems: Applications in Mechanics and Electronics. Springer, Berlin.

[4] Legrand M., Pierre C. (2022) Compact Weighted Residual Formulation for Periodic Solutions of Systems Undergoing Unilateral Contact and Frictional Occurrences. 10th European Nonlinear Dynamics Conference, Lyon, 17-22 July 2022.