Vibro-impact NES: Nonlinear mode approximation using the multiple scales method

Balkis Youssef and Remco I. Leine
Institute for Nonlinear Mechanics, University of Stuttgart, Stuttgart, Germany

Abstract. The aim of the paper is to derive a *closed-form* approximation for a nonlinear mode of a system with vibro-impact NES. Hereto, the multiple scales method is used to analyze the dynamical behavior of a linear oscillator coupled with a vibro-impact nonlinear energy sink (VI NES). The steady state response in the vicinity of 1:1 resonance is approximated. The resonance frequency of the examined nonlinear system for different excitation levels is estimated and the corresponding backbone curve is identified. The theoretical findings agree with the simulation results and represent a possible new approach for system identification.

Introduction

A nonlinear energy sink (NES) is a structural element, which is attached to a primary structure to absorb and mitigate the vibration energy, when the structure is excited within a certain frequency range. To assure an optimal design of the NES, many studies (e.g. [4, 3]) have been conducted to understand its working principle, being strongly related to the theory of nonlinear modes (NM). Different numerical approaches, including control-based methods, have been applied to approximate NMs of nonlinear vibrating structures. These methods are mainly based on Rosenberg's definition for nonlinear modes and rely on the invariance property of the NM in the configuration space under the system's flow [1]. In this work, the simplest case of periodic motion of the VI NES is considered, namely 1:1 resonance with 2 symmetric impacts per period. The method of multiple scales is applied to approximate the solutions and to derive a closed-form expression for the backbone curve. In this context, the backbone curve is defined as the curve connecting the maximas of the frequency response functions for different excitation levels [2].

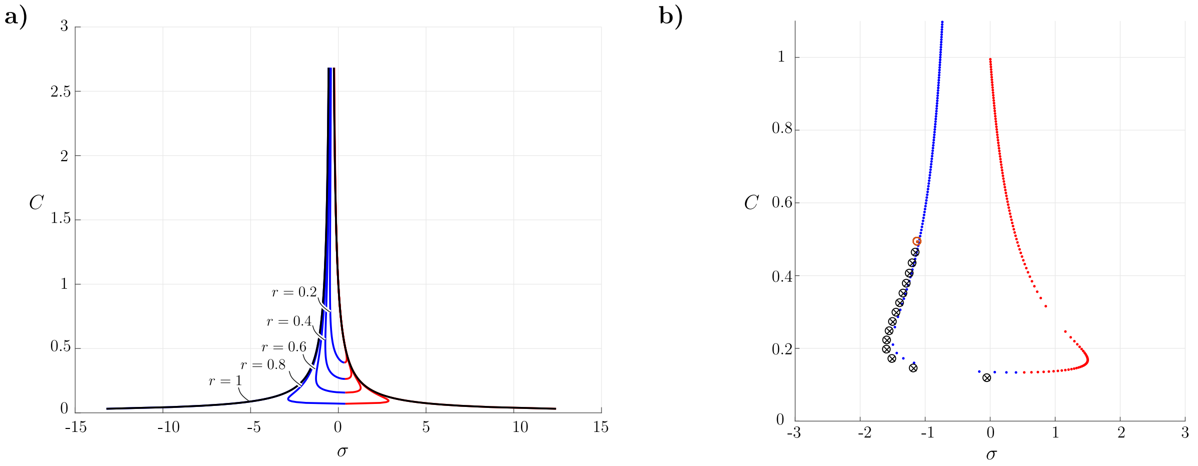

Figure 1: a) Backbone curves for different values of r. b) Backbone curve for $r = 0.65$. The red point on the left branch represents the point attained with PLL. The black crossed circles correspond to the estimated values from the free resonant decay response.

Results and discussion

The developed analytical treatement from [3] is pushed further to extract the modal properties of the studied system. A closed-form description of the system's considered backbone curve is derived and numerically verified. Figure 1 depicts the obtained curves for different values of the coefficient of restitution r.

In a second step, a verification of the obtained modal lines is persued via a numerical approach. The numerical identification is achieved through the system's resonance decay response combined with a Phase-Locked-Loop (PLL). At this stage, the backbone curve is obtained by means of the decaying free resonant response. The estimated modal line is shown in Figure 1, depicting the numerically identified stable branch of the backbone curve and confirming the analytical results.

References

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