Parametric optimization of fold bifurcation points

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Abstract. The aim of this work is to optimize the parameters of a mechanical system in order to force fold bifurcation points to appear at targeted frequencies. To this end, an original harmonic balance-based optimization procedure is developed. Functions similar to those employed during bifurcation tracking analyses are used to characterize fold bifurcations in the objective function. The proposed approach is illustrated on a Duffing oscillator with cubic nonlinearity.

Introduction

The ever-increasing demand for lighter structures and more efficient systems requires that the effects of nonlinearities be evaluated at the design stage. One of the most notable characteristic that sets apart nonlinear systems from their linear counterparts is bifurcation phenomena. When a parameter is varied, e.g. the forcing frequency, a bifurcation may occur, resulting in qualitatively different responses such as quasi-periodic or chaotic oscillations. Bifurcation analysis, which aims at predicting and studying such phenomena is a thriving research field. Recent research investigated the computation of the stability of periodic solutions [1] or the parametric analysis of bifurcation points [2, 3, 4]. However, very few studies [5] attempted optimizing the structural parameters of a mechanical system for it to exhibit bifurcations at desired locations and never in the context of nonlinear vibrations. In this work, we develop a computational optimization framework based on the harmonic balance method (HBM), which is widely used in the nonlinear mechanical vibration community. The formulation of the objective function for the optimization strategy relies on a bifurcation measure formulated via a bordering technique and Hill stability analysis and allows one to simultaneously handle multiple bifurcation without two being matched to the same target location.

Results and discussion

The proposed methodology is applied to a Duffing oscillator with hardening cubic nonlinearity. The initial forced response curve shown in Figure 1 (left) exhibits 8 bifurcations: 6 folds and 2 branch points. Two target locations for the fold bifurcations are defined as $\Omega_{tar,1} = 2$ and $\Omega_{tar,2} = 2.5$ rad/s and the optimization is carried out with respect to the damping coefficient. Figure 1 (right) shows the forced response curve computed with the optimized value of the damping coefficient. It is clear that the fold bifurcations located on the primary resonance match with the imposed target locations. One can also note that only two folds are present as the objective function is defined such that the minimum is reached when the number of detected bifurcations match the number of targeted locations [6]. These results are promising and give confidence for application on industrial test cases.

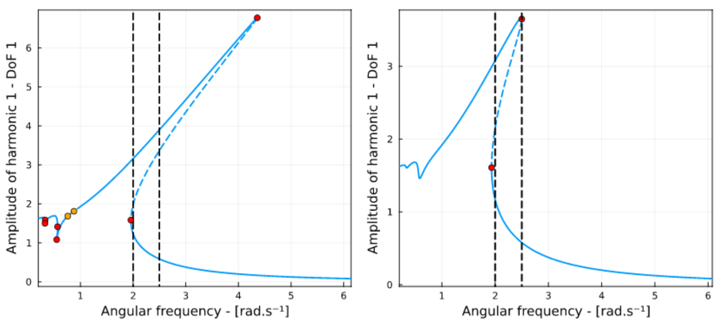

Figure 1: Forced response curve of the Duffing oscillator before (left) and after (right) optimization of the damping coefficient. Target locations $\Omega_{tar,1} = 2$ and $\Omega_{tar,2} = 2.5$ rad/s (vertical black dashed lines). Fold bifurcations and branch points represented by red and orange circle markers, respectively.

References

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