Robust topology optimization under non-probabilistic uncertainties

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Abstract. A novel approach to static and dynamic topology optimization has recently been proposed by the authors that aims to minimize a proper norm of the input/output transfer matrix, say **G**. The key ingredient is the Singular Value Decomposition (SVD) that is useful with respect to engineering point of view.

Introduction

This work presents a topology optimization approach that is innovative with respect to two distinct matters. First of all the proposed formulation is capable to handle static and dynamic topology optimization with virtually no modifications. Secondly, the approach is inherently a multi-input multi-output one, i.e. multiple objectives can be pursued in the presence of multiple loads. The method is based on the input-output mapping that is inherently algebraic in the static case and becomes such in the dynamic case thanks to the adoption of a frequency-domain framework. The Singular Value Decomposition (SVD) [1] of the resulting transfer function, say **G**, represents then the core of the proposed approach. Singular Value Decomposition is useful with respect to two different matters:

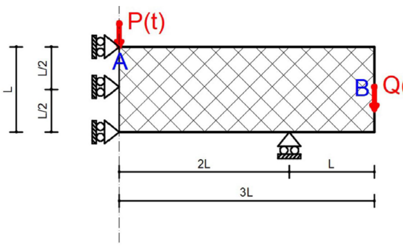
- norms used as goal functions may be uniquely defined in terms of the singular values of **G**. The sensitivity analysis may therefore be given a compact and clear format that was shown to work properly in statics as well as in dynamics [2];
- from an engineering point of view, any singular value is shown to be the gain (blow-up factor) of the associated input-output channel in which the system is decomposed. Singular values are therefore the inherent quantities that should enter any structural optimization formulation.

Results and discussion

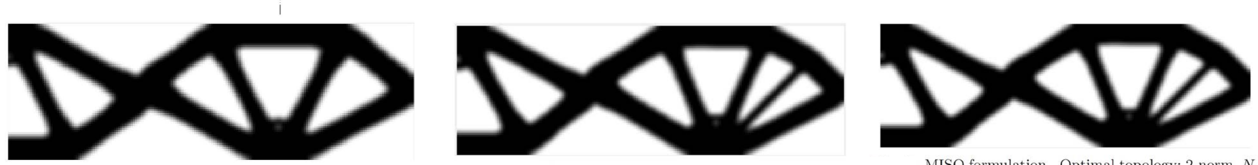
That said, the goals of this paper are the following ones:

- Load (and possibly material) uncertainties are added to the structural model to be designed and a formulation is proposed that is capable to handle such uncertainties and drive the procedure toward a robust optimal design within a worst-case scenario approach (see Fig. 1);
- toward a fully geometrically non-linear analysis, a preliminary approach is derived that allows to derive optimal topology with respect to buckling using a sequence of linearised problems, each of which is of the same type as the one introduced above.

Numerical examples concerning static and dynamic problems are presented.



$$
\begin{cases}
\min\limits_{\boldsymbol{p}} \mathscr{F}(\boldsymbol{p}) = ||\boldsymbol{G}(i\omega, \boldsymbol{p})|| \\
\text{s.t.} \qquad \boldsymbol{G}(i\omega;\ \boldsymbol{p}) = \boldsymbol{C}\,(i\omega \boldsymbol{E} - \boldsymbol{A})^{-1}\,\boldsymbol{B} \\
\qquad\qquad V(\boldsymbol{p}) \leq V_{\max} \\
\qquad\qquad \boldsymbol{p}_{\min} \leq \boldsymbol{p} \leq \boldsymbol{p}_{\max}
\end{cases},
$$



SISO formulation - Optimal topology: 2-norm, N-norm and F-norm

SIMO formulation - Optimal topology: 2-norm, N-norm and F-norm

MISO formulation - Optimal topology: 2-norm, N-norm and F-norm

Figure 1: Example of optimization in elastostatic condition: SISO - single load case and standard compliance minimization (left), SIMO - single load case and minimization of the compliance vector norm (center), MISO - two load cases and standard compliance minimization (right)

References

[1] K. Strang 2019 Linear Algebra and Learning from Data. Cambridge Press, Wellesley, Wellesley.

[2] Venini, P., Pingaro, M. (2023) Static and dynamic topology optimization: an innovative unifying approach. *Structural and Multidisciplinary Optimization* (accepted).