Dispersion properties of metamaterial honeycombs embedding nonlinear resonators

Yichang Shen[*] and Walter Lacarbonara[*]

[*]*Department of Structural and Geotechnical Engineering, SAPIENZA University of Rome, Rome, Italy*

Abstract. Wave propagation of honeycomb metamaterials hosting nonlinear resonators is studied. The projection method based on the Floquet-Bloch theorem is used to explore the linear and nonlinear dispersion properties. The method of multiple scales is applied to get the closed form expression of the nonlinear manifolds parametrized by the amplitudes, frequency, and wave number. The effects of the nonlinearity on the stop band frequency are thus investigated, furthermore, the optimization issues of the resonators' mass, stiffness, damping and nonlinearity towards certain requirements of the stop band characteristic are addressed.

Introduction

Mechanical metamaterials are attracting a great deal of attention thanks to their potential for suppressing or attenuating the propagation of elastic waves by creating stop bands. In the literature, many works addressing metamaterial focused on the linear dispersion properties or on the nonlinear frequency response, the present work aims to investigate the effects of the local nonlinear resonators parameters on the dispersion relations of the honeycomb (see the schematic view inside Fig. 1(a)), and thus, to ensure stop band behavior and to find the optimal design of the resonators. The analytical prediction of the stop band properties and the comparison between the generated nonlinear stop bands and the linear counterparts are investigated.

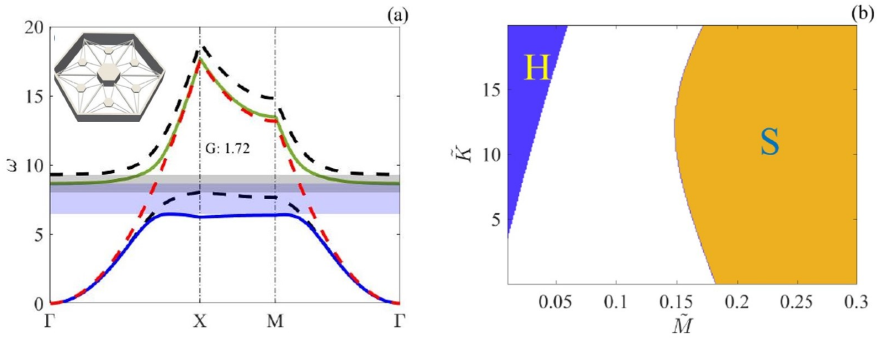

Figure 1: (a) The dispersion curves of the honeycomb with single-dof nonlinear resonators, the red and black dashed curves represent the dispersion properties of the plate without resonators and with linear resonators, respectively. The nonlinear optical and acoustic frequencies are shown by the blue and green curves, respectively. G is the ratio between nonlinear (blue) and linear (grey) stop band width. (b) Design chart with the suggested type of resonator nonlinearity based on the stiffness and mass of resonators.

Results and discussion

The linear dispersion functions ω^- and ω^+ are obtained by the eigenvalue problem of the mechanical system [1], which describe the lower-frequency curve (acoustic branch) and the higher-frequency curve (optical branch) of the linear dispersion spectrum, respectively. By applying the asymptotic approach of multiple scales [2] to the equation of motion in modal coordinates, one can finally obtain the nonlinear frequency of the acoustic mode (ω_{nl}^-) and optical mode (ω_{nl}^+), read as:

$$\omega_{nl}^- = \omega^- + \frac{N_3(3{\phi_2^-}^4 {a^-}^2 + 6{\phi_2^-}^2{\phi_2^+}^2{a^+}^2)}{8\omega^-}, \quad \omega_{nl}^+ = \omega^+ + \frac{N_3(3{\phi_2^+}^4{a^+}^2 + 6{\phi_2^+}^2{\phi_2^-}^2{a^-}^2)}{8\omega^+}, \qquad (1)$$

where $a^\pm$ are the modal coordinates and $\phi^\pm$ are the eigenvectors depend on the resonator's mass and stiffness, N_3 is the resonators nonlinearity. Figure 1(a) shows linear and nonlinear dispersion functions of the honeycomb, it should be noted that the stop band width increases up to 72% when the softening nonlinearity is involved. Figure 1(b) represents a design chart showing the optimal choice of hardening or softening resonators based on the their mass and stiffness to enlarge the stop band width [3]. In summary, an in-depth understanding of the nonlinear metamaterial's dispersion properties and the possibility provided by the exploitation of the resonator nonlinearity to improve the metamaterial's vibration suppression capability can be gained from the numerical sensitivity optimization studies carried out within the analytical computations of the nonlinear wave dispersion properties.

References

[1] Y. Shen, W. Lacarbonara, (2023) Nonlinear dispersion properties of metamaterial beams hosting nonlinear resonators and stop band optimization. *Mech. Syst. Signal Process* **187**:109920.

[2] A. H. Nayfeh, D. T. Mook, (1979), Nonlinear oscillations, John Wiley & sons, NY.

[3] Y. Shen, W. Lacarbonara, (2023) Nonlinearity-enhanced wave stop bands in honeycombs embedding spider web-like resonators, *J. Sound Vib.* (Under review).