An enhanced pathfollowing scheme for nonsmooth dynamics via improved computation of the monodromy matrix

Giovanni Formica*, Franco Milicchio** and Walter Lacarbonara***

* *Department of Architecture, Roma Tre University, Italy, ORCID #0000-0002-6410-8083*
** *Department of Engineering, Roma Tre University, Italy, ORCID #0000-0002-4875-4894*
*** *Department of Structural and Geotechnical Engineering, Sapienza University of Rome, Italy, ORCID #0000-0002-8780-281X*

Abstract. One of the key features of pathfollowing schemes for non-autonomous nonlinear dynamical systems is the computation of the Jacobian of the Poincaré map (*i.e.*, the *monodromy matrix*), which, being employed as iteration matrix, governs both the accuracy and robustness of the whole numerical strategy. In the context of nonsmooth nonlinear dynamic systems, this matrix can only be computed via numerical differentiation, which leads to subtractive cancellation errors. By means of a wide numerical campaign for meaningful multi-DOFs systems, we discuss strategies to reduce such errors via an enhanced pathfollowing scheme, recently proposed by the authors.

Introduction

Exploring manifolds of solutions of nonlinear dynamical problems with enhanced continuation algorithms is attracting high scientific and technological interest, especially when handling both accurately and efficiently large-scale engineering systems [1]. We focus on pseudo-arclength continuation approaches searching numerically for the fixed points of a Poincaré map $\mathbf{P}$ of nonlinear dynamical systems along the curve of the corresponding periodic solutions. The cornerstone of such approaches when facing nonsmooth dynamical systems (*e.g.*, systems with hysteresis and discontinuities of various kinds) is the construction of the Jacobian matrix involving the computation of the monodromy matrix $\mathbf{H}$. The central difference scheme used to compute $\mathbf{H}$ leads to cumulative errors, which turn out to be the combination of the errors introduced by using the small finite-difference coefficient with the errors affecting the time integration to obtain $\mathbf{P}$. Here we present an upgrade of the pseudo-arclength numerical strategy proposed in [2], and developed in a `C++` software library (freely available at `https://zenodo.org/record/7245478`). The strategy works with an accelaration procedure, based on a Krylov subspace loop, nested in a modified Newton-Raphson scheme using $\mathbf{H}$ as iteration matrix.

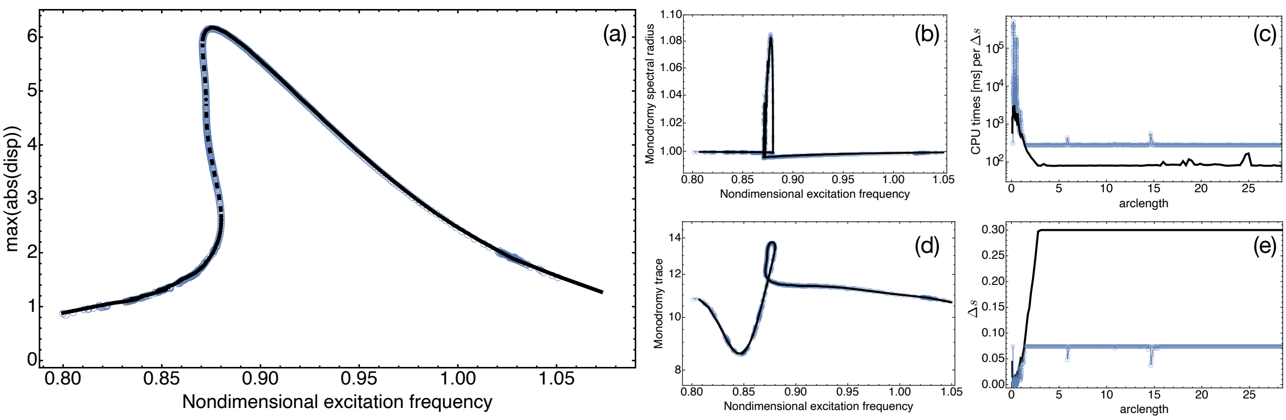

Figure 1: (a) frequency response curves and (b, d) monodromy invariants (spectral radius and trace, respectively) for hysteretic 10-dof system; (c) CPU times per arclength step-size Δs and (e) Δs along the arclength of the solution path (SP solid line, CD dotted circles).

Results and discussion

The proposed numerical scheme works as a Runge-Kutta single-pass (SP) method, where a numerical differentiation computed by Lánczos integration is directly employed within the time step integration yielding the monodromy matrix. This makes the small finite-difference coefficient act on a time-discretization error which is one-order-of-magnitude greater than the time-discretization error affecting the integration of the perturbed Poincaré map as in standard central difference (CD) schemes. We here report the results concerning a 10-dof system (30-dim state problem) featruing hysteretic nonlinear springs and linear dashpots which connect masses in a parallel arrangement. We compare the performance of the strategy using the CD scheme with that employing the proposed SP scheme. The latter proves to be not only more accurate but also more robust than the former, leading to a faster convergence. Figure 1(e) shows how the SP scheme allows to achieve convergence employing an arclength step size Δs of the solution path 4 times larger than the standard CD scheme, and the CPU time per Δs in SP is about 3.5 lower than the CPU time required by CD, see Figure 1(c).

References

[1] Ahsan Z., Dankowicz H., Li M., Sieber J. (2022) *Nonlinear Dyn* **107**:3181–3243.
[2] Formica G., Milicchio F., Lacarbonara W. (2022) *Int J Non-Linear Mech* **145**:104116.