Diagrammatic perturbation theory for Stochastic nonlinear oscillators

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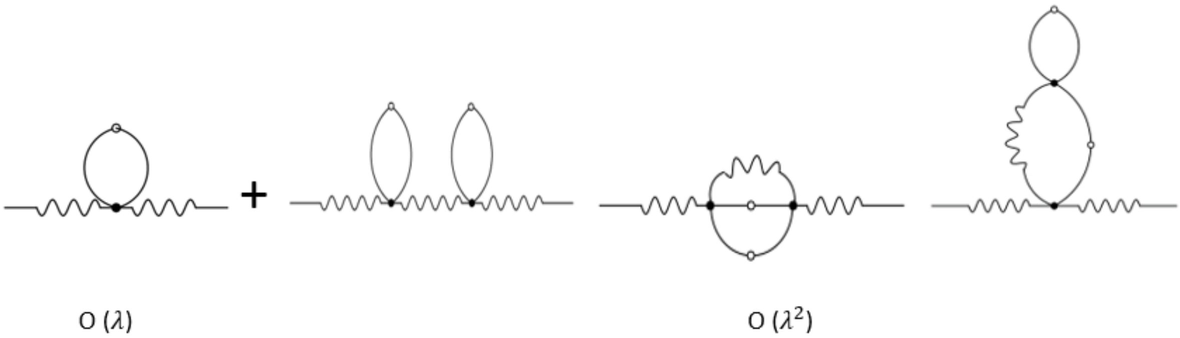
Abstract. In this work we consider stochastic driven damped nonlinear oscillator, characterized by a natural frequency ω_0 and a damping coefficient 2Γ of the linear system. We explore what the time averaged linear response of the nonlinear system will be in the frequency domain. We show that a perturbation theory involving the nonlinear terms is identical to the Feynman diagrams based perturbation theory used in any statistical or quantum field theory. We find that at second order the frequency and the damping coefficient becomes dependent on the frequency ω at which the system is being probed. This is a small but unexpected effect.

Introduction

The noise driven nonlinear oscillator is described by the dynamics:

$$\ddot{x} + 2\Gamma\dot{x} + \omega_0^2 x + \lambda x^3 = f(t) \tag{1}$$

where $f(t)$ is a random Gaussian white noise with the two point correlator given by $\langle f(t_1)f(t_2)\rangle = 2D\delta(t_1 - t_2)$ where D is a constant. For the sinusoidally driven case, we have an additional $Acos(\Omega t)$ term on the right hand side. The frequency dependent linear response of the system is given by $R(\omega) =< \frac{\delta x(\omega)}{\delta f(\omega)} >$ where the angular bracket is a long time average.For $\lambda = 0$, $R(\omega) = R_0(\omega) = (\omega_0^2 - \omega^2 + 2i\Gamma\omega)^{-1}$. At the first order in perturbation theory the addition to ω_0^2 is $\Delta\omega_0^2 = \frac{3\lambda D}{2\omega_0^2}$ which is in agreement with Samanta et al[1,2] and the change $\Delta\Gamma$ in the damping is zero. Our perturbation framework is based on the diagrammatic approach initiated for stochastic dynamics by Kraichnan [3] and Wyld [4].Our approach is complementary to the recent work of Belousov et al [5]. It allows for a straightforward calculation of the $O(\lambda^2)$ term and we find that the correction to ω_0^2 is frequency dependent and more importantly there is a non-zero $\Delta\Gamma$ which is also frequency dependent. Both corrections vanish at very high frequencies which is physical.



Feynman diagrams of O(λ) and O(λ^2)

Results and discussion

We extend this framework to address the stochastic pendulum with a period forcing $Acos(\Omega t)$ (an additional contribution to the R.H.S of Eq.(1). The shift in the response function contains joint contribution of the stochastic and deterministic drives i.e AD.Further, if we replace the λx^3 term by μx^2 in Eq.(1), then we have a metastable cubic potential well and our perturbation approach allows us to find the criterion for which the particle can escape from the well. The result we get is analogous to fluctuation dissipation theorem in statistical mechanics. We can also handle the stochastically driven Kapitza pendulum.We find that the stability of the inverted fixed point is possible if we have colored noise, rather than white noise [6].

References

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