Travelling wave and Turing patterns in subdiffusive autocatalytic system

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Abstract. We analyse two-variable cubic autocatalytic reactions in a porous media which are governed by subdiffusion. The mean square displacement of molecules here scale as t^{γ}, where $0 < \gamma < 1$. This system is governed by fractional partial differential equations. The system has multiple steady states. We investigate the effect of subdiffusion on the stability of the steady-states of the system. We use singularity theory to identify critical surfaces across which the bifurcation diagrams vary. The system exhibits travelling wave solutions connecting two steady states. This has been obtained from phase plane analysis analytically. In addition, the system also shows Turing patterns in selected regions of parameter space. This approach is based on identifying different critical surfaces across which steady-state stability changes when diffusive effects are included. The behaviour in the different regions is verified by an implicit numerical method based on the $L1$ scheme.

Introduction

Two commonly exhibited spatio-temporal patterns observed in natural systems are: travelling wave solutions and Turing patterns [1]. These patterns arise from the interaction of nonlinear reaction and diffusion and have been intensively studied in the literature [2]. Chemical species such as macromolecules or polymers often encounter molecular crowding due to their large size. In porous media, these molecules get trapped in pores and their transport is characterised by anomalous subdiffusion [3, 4]. Our system is a two variable cubic autocatalytic system in a porous media that is governed by fractional partial differential equation.

Results and Discussion

The fractional reaction subdiffusion system of two variables for autocatalytic kinetics has been analysed. We have used a numerical method based on the $L1$ scheme to solve the system. Our scheme is implicit and robust and has been used to predict travelling wave solutions and Turing patterns.

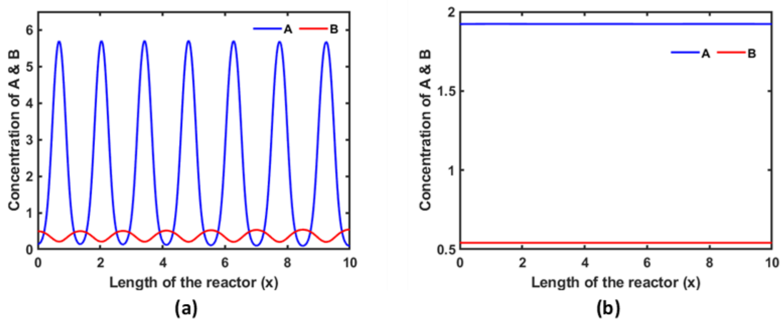

Figure 1: Profiles of species A and B with spatial variation at steady state for parameter: $d_r = 0.01$, $g_b = 0.15$, $b_b = 2.0$, $p_a = 0$, $b_b = 3$ and $\gamma = 0.8$. a) Turing patterns for $d_r = 0.01$, and b) Spatially homogeneous solution for $d_r = 0.09$.

The interaction of nonlinear kinetics and subdiffusion results in the travelling wave patterns. These waves are responsible for the rapid transport of species. The critical surface boundary has also been obtained by linear stability analysis. The fractional order system also exhibits Turing patterns as shown in Fig. 1(a) when the ratio of diffusivity of activator to inhibitor is lower than a critical value. Above this value, there is spatially homogeneous profile of species as shown in Fig. 1(b). We also plotted the Turing pattern for a different fractional power of the derivative and observed that the steady state behaviour does not depend on the fractional derivative. The fractional reaction subdiffusion model can be advantageous in studying biological systems where species transport is hindered by traps, macromolecular crowding, or other obstacles. The fractional power of derivative can be considered as a parameter that depends on pore size distribution for a particular system.

References

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