# Variable Length Control of a Planar Pendulum with Time Averaged Constant Cable Length

**H. Joseph***, M. Lanzerotti** and W. Lacarbonara*

*Dipartimento Ingegneria Strutturale e Geotecnica, Sapienza University of Rome, Rome, Italy ORCID #

**Department of Electrical and Computer Engineering, Virginia Polytechnic Institute and State University, Blacksburg, VA. 24061. U.S.A., ORCID # 0000-0001-7802-1117

**Abstract**. This work proposes the control of oscillations of a pendulum while maintaining a net constant length of the pendulum where the average is taken over several oscillations. As in prior work, the pendulum is shortened at the end of the swing and lengthened in the middle of the swing [1]. In this work, the pendulum is lengthened the same amount as it is shortened so that the net pendulum length remains unchanged as averaged for several oscillations. The shortening and lengthening can be accomplished with uniform shortening and lengthening and also with a hoisting control method with variable cable length [1].

## Introduction

We propose a control method to stabilize a payload in medical evacuation rescues [1]-[5]. We consider an operational aspect of medical evacuation (MEDEVAC) rescues with the aim to stabilize the swing angle of an oscillating rescue while maintaining the cable length when averaged over multiple oscillations [1].

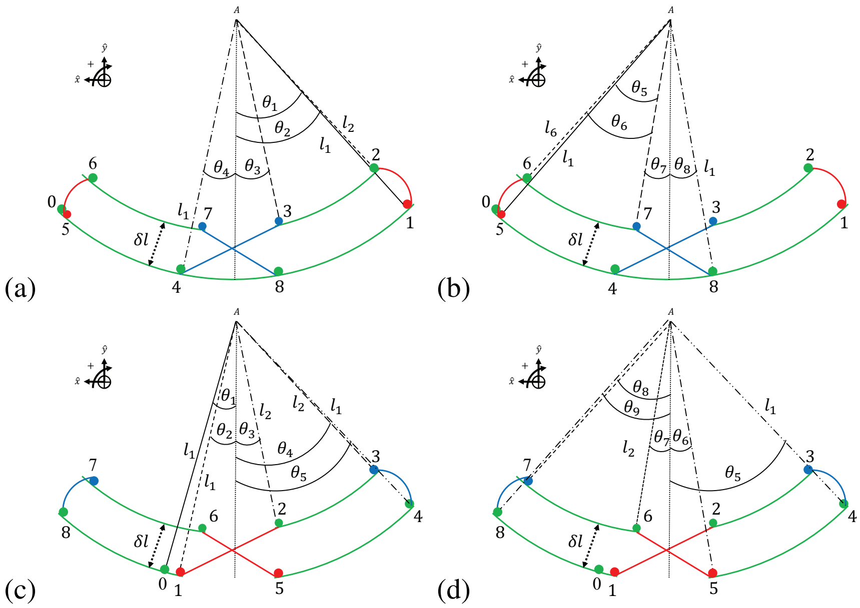

Figure 1: Method to decrease the swing angle of a pendulum while maintaining constant length $l_1$ in (a) steps 1-4 and (b) steps 5-8; Method to increase swing angle of a pendulum while maintaining constant length $l_1$ in (c) steps 1-4 and (d) steps 5-8.

## Results and Discussion

The proposed method considers the planar pendulum under gravity, $g$, with the second order nonlinear equation of motion such that $\ddot{\theta}+2\frac{\dot{l}(t)}{l(t)}\dot{\theta}+\frac{g}{l(t)}\theta=0$. The length of the cable can be shortened at the ends of the swing (Fig. 1(a)). Some choices of time dependence are: (i) $l(t)=l_0-v_s t$; (ii) $l(t)=l_a\,e^{-r(t\,-\,t_a)}+l_b-l_b\,e^{-r(t\,-\,t_a)}$ with $l_a>l_b$ (exponential shortening); (iii) $l(t)=l_a+(l_b-l_a)\Big[\frac{1}{2}+\frac{1}{2}\tanh\Big(r(t-t_{a,b})\Big)\Big]$ with $l_a>l_b$. The cable can also be lengthened at the middle of each swing (Fig. 1(b)). Some choices of time dependence are: (iv) $l(t)=l_0+v_l t$; (v) $l(t)=l_e e^{-r(t-t_e)}+l_f-l_f e^{-r(t-t_e)}$ with $l_e>l_f$ (exponential lengthening); (vi) $l(t)=l_e+(l_f-l_e)\Big[\frac{1}{2}+\frac{1}{2}\tanh\Big(r(t-t_{e,f})\Big)\Big]$ with $l_e<l_f$.

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