Nonlinear vibration control of a slightly curved beam with distributed piezoelectric patches

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Abstract. This paper aims to study the effect of axial forces and bending moment on the static behaviour and natural transversal vibrations of piezoelectrically actuated slightly curved beams subject to prescribed axial end displacement. This is achieved by bonding patches of piezoceramic material at along the top and bottom surfaces of the beam. The governing equations for the system are formulated using Euler-Bernoulli beam theory and on the basis of von Kármán non-linear strain–displacement relations and linear constitutive relations for both the piezoelectric and host beam materials. Due to the nonlinearity of the governing equations, their solutions are attempted by a regular perturbation technique leading to asymptotic expansions of displacements, internal forces and the natural vibration frequency. Numerical examples are presented, including how the geometrical nonlinearity is incorporated in the system's static and dynamic responses as a function of two main variables, the beam initial curvature and the electric field application.

Problem statement

Statics and dynamics of composite structures with geometrical imperfections actuated piezoelectrically or electrostatically have aroused great interest due to their various potential applications [1-3]. In this work a simply supported sandwich piezoelectric beam of length l and width b, composed of two identical external piezoelectric layers (thickness h_p) perfectly bonded to an elastic core of thickness h has been considered. The adhesive layers are neglected. The beam of initial curvature described by function W_0(x) is subjected to a prescribed end displacement δ that changes the distance between the pin supports, as shown in Fig. 1a. The geometry of the beam cross-section is given in Fig. 1b.

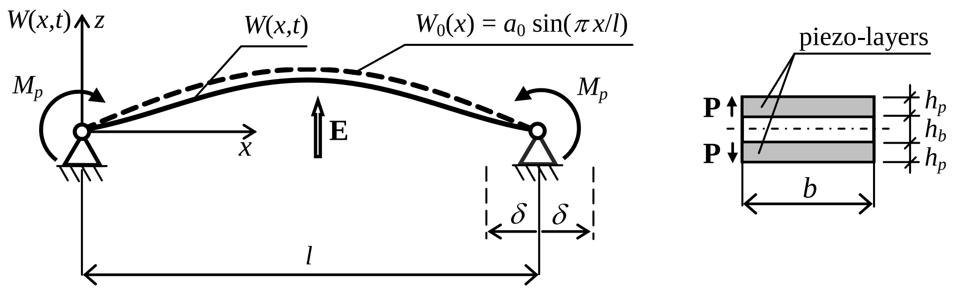

Figure 1: Scheme of a simply supported slightly curved piezoelectric beam and its cross-section.

Since the piezo-layers have an opposite polarization, the application of the electric field of vector **E** makes that the upper layer is contracted, whereas the bottom one is expanded and, as a result, the piezoelectric moments of equal values M_p appear at the supports. Moments M_p may be redirected when vector **E** will have the opposite direction. As both beam ends are pinned, the beam deflection influences an axial force that is caused by the prescribed displacement. The analysis is based on a nonlinear equation of motion of a shallow beam under both axial and piezoelectric forces. The perturbation method of the solution has been chosen according to which the transversal and axial displacements, the axial forces and the natural vibration frequency are expanded into power series with respect to a small amplitude parameter. Infinite sets of differential equations with associated boundary conditions are obtained for the rising power of the amplitude parameter which are solved sequentially. The first set of equations describes the static response of the system presenting the effect of the axial force and the applied voltage on the static equilibria of the beam. The next set of equations represents the eigenvalue problem for various static equilibrium positions. A large number of results presents the variations of the natural frequencies and mode shapes for equilibrium positions of the beam being stretched or compressed by axial forces resulting from the applied voltage changing from negative to positive magnitudes. The results have also shown the possibility of exploiting electric voltage to tune the stability and natural frequencies of curved beams over a wide range of frequencies.

References

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