# Towed wheel shimmy suppression via a nonlinear tuned mass damper

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**Abstract.** The implementation of a nonlinear tuned mass damper (TMD) to suppress shimmy vibrations in towed wheels is addressed. Stability analysis illustrates that the TMD can significantly improve the stability of the equilibrium of the wheel. Besides, bifurcation analysis revealed that adding a softening stiffness in the TMD restoring force enables the suppression of periodic solutions coexisting with the stable equilibrium for a classical linear TMD.

## Introduction

Shimmy is a common name for lateral-torsional vibrations of wheels. This is a typical stability problem in vehicle dynamics, affecting several types of vehicles [1]. The vibrations are generated by the dynamically varying forces in the tire-road contact, and they can arise at various speeds. This study aims to assess the performance of a nonlinear tuned mass damper (TMD) for the suppression of shimmy instabilities.

## Results and discussion

Intending to provide a general understanding of the performance of a TMD, we adopt the rather simple tire model provided by Pacejka's Magic Formula [2]. The suspension is modeled as a one-degree-of-freedom (DoF) system, while the TMD is attached to it and can rotate around the axis of rotation of the suspension. The following system of differential equations models the dynamics of the mechanical system:

$$
\begin{aligned}
&I_t\ddot{\psi} + c_\psi\dot{\psi} + k_\psi\psi + c_a\left(\dot{\psi} - \dot{\varphi}\right) + k_a\left(\psi - \varphi\right) + k_{nl}\left(\psi - \varphi\right)^3 = M_z - eF_y \\
&\sigma\dot{\alpha}' - V\cos(\psi)\alpha' = V\sin(\psi) + (e-a)\,\dot{\psi} + \sigma\alpha'^2\psi \\
&I_a\ddot{\varphi} + c_a\left(\dot{\varphi} - \dot{\psi}\right) + k_a\left(\varphi - \psi\right) + k_{nl}\left(\varphi - \psi\right)^3 = 0,
\end{aligned}
\tag{1}
$$

where $\psi$ is the yaw angle of the wheel, $\varphi$ is the rotation angle of the TMD, $\alpha'$ is the tire deformation angle at ground level, and $V$ is the normalized towing velocity. Physical interpretation of the other parameters is provided in [2, 3]. System parameters are representative of a non-dimensionalized aircraft landing gear [4].
A comparison between the stability chart without and with the TMD is provided in Fig. 1a,b. The TMD is optimized for mechanical trail $e = 1$, for which the maximal unstable velocity is reduced from 89.7 to 6.5. A bifurcation analysis illustrated that Hopf bifurcations mark stability boundaries. For a linear TMD ($k_{nl} = 0$), the right bifurcation exhibits a strongly softening character, generating a large region of bistability, which significantly reduces the region of safe operation (Fig. 1c, black line). Introducing a small softening nonlinearity the bistable region can be completely suppressed, also reducing LCO amplitude (Fig. 1c, purple line). Softening stiffness also partially destabilizes LCOs through a pair of Neimark-Sacker bifurcations, generating quasiperiodic and chaotic motions. However, their amplitude is not significantly larger than that of LCOs, which makes them not particularly harmful from an engineering perspective.

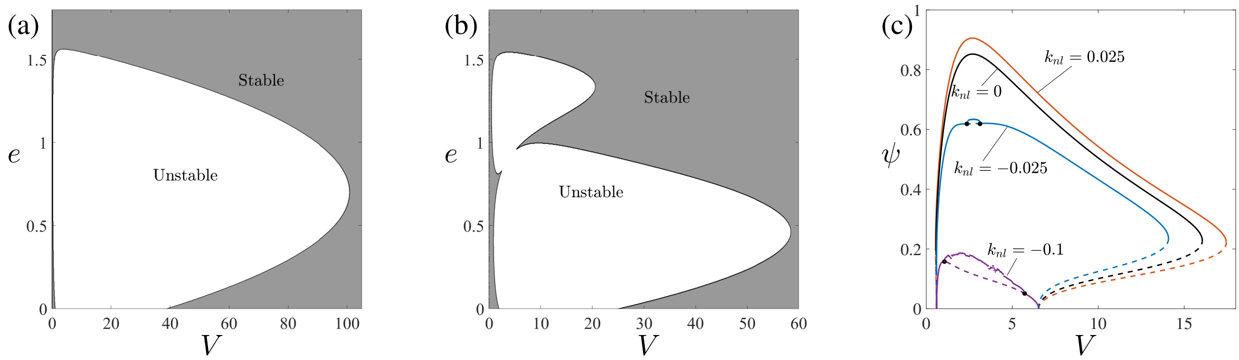


Figure 1: Stability chart without (a) and with (b) TMD, optimized to minimize unstable region for $e = 1$; (c) bifurcation diagram for different values of the absorber nonlinear stiffness $k_{nl}$, black dots indicate Neimark-Sacker bifurcations.

## References

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