Persistence of periodic orbits from piecewise linear near-hamiltonian differential systems having a two saddle and one center

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Abstract. In this paper, we study the number of limit cycles that can bifurcate from a periodic annulus of discontinuous planar piecewise linear Hamiltonian differential system with three zones separated by two parallel straight lines, such that the linear differential systems, that define the piecewise one, has saddles and centers. More precisely, the central subsystem, i.e. the linear system in the region bounded by the straight lines, has a saddle and the other two have a saddle and a center, respectively. We prove that the maximum number of limit cycles that bifurcate from a periodic annulus of this kind of piecewise Hamiltonian differential systems, by linear perturbations, is at least five. For this, we obtain normal forms for the systems and study the number of zeros of its Melnikov functions defined in two and three zones.

Introduction

The study of the number and position of limit cycles is the main problem of the Qualitative Theory of ODEs. Currently this study has been considered for piecewise differential systems in regions separated by a straight line. These systems appear in a natural way in mechanics, electrical circuits, control theory, neurobiology, etc (see the book [1]). Recently, piecewise differential system defined in regions with more than two zones have attracted the attention of researchers. When the central subsystem has a center at the origin and the others subsystems have centers or saddles, then the maximum number of limit cycles is at least three, see [2]. Now, when the central subsystem has a saddle at the origin and the others subsystems have virtual centers then the maximum number of limit cycles is at least five, see [3].

Main result

In this paper, our goal is estimated the lower bounds for the number of crossing limit cycles of a discontinuous piecewise linear near-Hamiltonian differential systems with three zones, given by

$$\left\{ \begin{array}{l} \dot{x} = H_y(x,y) + \epsilon f(x,y), \\ \dot{y} = -H_x(x,y) + \epsilon g(x,y), \end{array} \right. \tag{1}$$

with $H(x,y) = \dfrac{b_j}{2}y^2 - \dfrac{c_j}{2}x^2 + a_j xy + \alpha_j y - \beta_j x$, $f(x,y) = r^j_{10}x + r^j_{01}y + r^j_{00}$, $g(x,y) = s^j_{10}x + s^j_{01}y + s^j_{00}$, for $(x,y) \in \Sigma^j$ where $j = R, C, L$, $\Sigma^L = \{(x,y) \in \mathbb{R}^2 : x \leq -1\}$, $\Sigma^C = \{(x,y) \in \mathbb{R}^2 : -1 \leq x \leq -1\}$, $\Sigma^R = \{(x,y) \in \mathbb{R}^2 : x \geq 1\}$, the dot denotes the derivative with respect to the independent variable t, here called the time, and $0 \leq \epsilon << 1$. When $\epsilon = 0$ we say that system (1) is a piecewise Hamiltonian differential system.

Let us assume that system $(1)|_{\epsilon=0}$ satisfies the following hypotheses:

(H1) The unperturbed central subsystem from $(1)|_{\epsilon=0}$ has a saddle at the origin and the others unperturbed subsystems from $(1)|_{\epsilon=0}$ have centers or saddles.

(H2) The unperturbed system $(1)|_{\epsilon=0}$ has only crossing points on the straights lines $x = \pm 1$, except by some tangent points.

(H3) The unperturbed system $(1)|_{\epsilon=0}$ has a periodic annulus consisting of a family of crossing periodic orbits such that each orbit of this family passes thought the two or three zones with clockwise orientation.

Thus, the main result of this paper is the follow.

Theorem 1 *The maximum number of limit cycles of system* (1)*, satisfying hypotheses* (Hi)*, for* $i = 1, 2, 3$*, which can bifurcate simultaneous from the periodic annulus of the unperturbed system* $(1)|_{\epsilon=0}$ *is at least five. More precisely, we have either at least 2 limit cycles passing through the three zones and 3 passing through two zones or 1 limit cycles passing through the three zones and 4 passing through two zones.*

References

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