Nonlinear vibration of small size beams on fractional visco-elastic foundation

Nikola Nešić [*], Milan Cajić[**], Danilo Karličić[**] and Julijana Simonović[***]
[*]*Faculty of Technical Sciences, University of Priština, Kosovska Mitrovica, Serbia*
[**]*Mathematical Institute of Serbian Academy of Science and Arts, Belgrade, Serbia*
[***]*Faculty of Mechanical Engineering, University of Niš, Niš, Serbia*

Abstract. This paper investigates the dynamic behaviour of a geometrically nonlinear nanobeam resting on the fractional visco-Pasternak foundation and subjected to dynamic axial and transverse loads. The nonlinearity is introduced into the model with the von Karman strain-displacement relation. The equation of motion is derived with Hamilton's principle, discretized with Galerkin's principle, and solved by using three different methods: perturbational multiple scale method, incremental harmonic balance method, and Newmark method. The cases with weak and strong nonlinearity are studied. The influence of different parameters, such as small scale parameters, external excitation, parameters of fractional visco-elastic foundation, etc., on amplitude-frequency diagrams is discussed in detail.

Introduction

Dynamic analysis of nanostructures under different excitation conditions is important for their application in nanoengineering practice. Introducing size and dissipation effects into continuum models of nanoscale structures can lead to more accurate results when compared to experiments and atomistic models. This paper analyses the transversal vibration of nonlocal [1] and nonlocal strain gradient [2] Euler-Bernoulli beam resting on a fractional visco-elastic foundation. The general nonlocal strain gradient constitutive relation is

$$(1-\mu^2\nabla^2)t_{xx} = (1-l^2\nabla^2)E(z)\varepsilon_{xx}. \tag{1}$$

When $l = 0$, it reduce to nonlocal theory. The nonlinearity is introduced into the model with the von Karman strain-displacement relation. Restitutive force of viscoelastic foundation is defined as:

$$F_m(x) = (k_w + K_w D^\alpha)w - (k_g + K_g D^\alpha)\frac{\partial^2 w}{\partial x^2}, \tag{2}$$

The parameters k_w and k_g are usually set to zero [1], however, we explore their influence with this analysis. Equations of motion are derived with Hamilton's principle, discretized with Galerkin approach, and solved by using three different methods: perturbational multiple scale method, incremental harmonic balance method, and Newmark method. The system dynamics are presented in the form of amplitude-frequency response plots.

Results and discussion

The influence of different parameters, such as small scale parameters, external excitation, parameters of fractional visco-elastic foundation, etc., on amplitude-frequency diagrams is analysed in detail. Results have shown that small parameters coming from stress gradient and strain gradient theory have a minor influence on the amplitude-frequency response. On the other hand, fractional derivative, external excitation, and visco-Pasternak foundation parameters have a significant impact on the response. Additionally, when considering a beam composed of functionally graded material, it has been shown that the power-law index displays a significant effect on the amplitude-frequency response. The system vibration amplitudes are higher for the odd values of the power-law index compared to materials with the even values of this parameter.

References

[1] Nešić, N., Cajić, M., Karličić, D., Janevski, G. (2021). Nonlinear superharmonic resonance analysis of a nonlocal beam on a fractional visco-Pasternak foundation. *PIME, Part C: J Mech Eng Sci* **235**(20):4594-4611.

[2] Nešić, N., Cajić, M., Karličić, D., Obradović, A., Simonović, J. (2022). Nonlinear vibration of a nonlocal functionally graded beam on fractional visco-Pasternak foundation. *Nonlin Dyn* **107**(3):2003-2026.