Modified Energy-based Time Variational Methods for Obtaining Periodic and Quasi-periodic Responses

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Abstract. Harmonic Balance Method (HBM) and Time Variational Method (TVM) are two semi-analytical methods that are used for solving periodically excited dynamical systems. Harmonic Balance Method is modified and extended in several works for solving free vibrations and systems with quasi-periodic responses. It is very important to study the homogeneous response to characterize the system with its features like natural frequencies and energy content. In this study, TVM is modified with the addition of energy equations for obtaining multi-frequency responses for multi-DOF autonomous systems, and these are applied to Duffing oscillator and 2-DOF pendulum system.

Introduction

Steady-state response of dynamical systems can be obtained using semi-analytical methods. Among these methods, Harmonic Balance Method[1] approximates the solution as a truncated Fourier series. Fourier series being periodic in nature, this method is an excellent tool used for obtaining limit cycles. This method is extended for multi-frequency excitation, called Multi Harmonic Balance Method (MHBM) and for homogeneous systems, called Harmonic Energy Balance Method (HEBM). One major drawback of these methods is its domain transformation method for the non-linear part of the system, which is a time-consuming process. Rook[2] introduced Time Variational Method, which skips the domain transformation step and solves the system in the time domain itself. Dhar and Krishna[3] extended this method similar to MHBM and obtained frequency-amplitude responses for quasi-periodically excited Duffing oscillator. In the beginning of this paper, the existing TVM and MTVM are described and applied to Duffing system. Cubic spline function is used as the basis function. The use of such a basis function requires modification in the concept of domain transformation in TVM, which is explained. In the following sections, the modified TVM is extended for computing periodic and quasi-periodic solutions for homogeneous systems named TVEM (Time Variational Energy based Method) and MTVEM (Multi Time Variational Energy based Method), respectively.

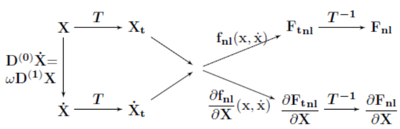

Figure 1: TVM modified with domain transformation

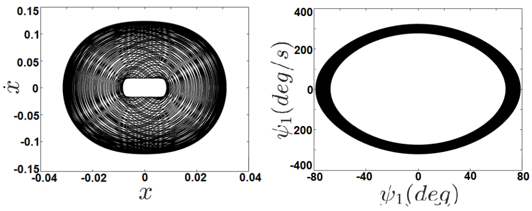

Figure 2: 2-DOF pendulum phase portraits x and ψ

Methodology and Result

TVM method is modified by introducing domain transformation shown in figure 1. The transformation T, being a linear map, is fast and does not produce any error in the process. This modified TVM is augmented with the Hamiltonian of the system forming TVEM. MTVM is modified using the Hamiltonian equation and the concept of energy split to obtain solutions at constant energy and different energy ratios in between the DOFs. TVEM and MTVEM are applied to the Duffing oscillator and pendulum system. Figure 2 shows result of the 2-DOF pendulum system (Translational DOF x and rotational DOF ψ) with total energy $\mathtt{E} = 4$ and energy ratio of the first DOF $\mathtt{E}_1/\mathtt{E} = 0.12$ solved using MTVEM and validated with numerical integration and MHEBM.

References

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