Analytical modelling for vibration analysis of partially cracked S-FGM plate

Dr Ankur Gupta*

Department of Mechanical and Aerospace Engineering, National Institute of Technology Raipur, GE Road Raipur, Chhattisgarh, INDIA 492010

Abstract. A nonlinear analytical model for vibration analysis of a partially cracked thin S-FGM plate affected by a partial crack is presented. The proposed analytical model is based on Kirchhoff\'s classical plate theory. The material properties of the S-FGM plate are assumed to vary according to the sigmoid function (two-power law distribution) of the volume fraction of the constituents. A single partial continuous line crack located at the centre is considered for analysis. The line spring model incorporates the effect of cracks in the form of moment and in-plane forces. The use of Berger\'s formulation introduces the nonlinearity in the model, and Galerkin's method is used to obtain the solution of the governing equation. Results for the fundamental frequencies as affected by the crack length and gradient index are presented for simply supported boundary conditions. It is found that the presence of a crack affects the fundamental frequencies of the partially cracked S-FGM plates.

Introduction

FGM plates are widely used in various mechanical, civil, and aerospace engineering applications. FGM plate is made from a mixture of ceramic and metal whose compositions are varied according to the required performance. An exhaustive review of recent developments in functionally graded (FG) plates can be found in the work of Jha et al. [1]. Sigmoid FGM (S-FGM) was proposed by Chung and Chi [2] in which two power-law functions are required to define a new volume fraction. The plate, an elementary structural element, often undergoes dynamic loadings and suffers failures due to unwanted vibrations; hence, vibration analysis of such structures must be carried out to ensure safety and smooth operation. Rice and Levy [3] used classical Kirchhoff's plate theory and proposed the line spring model (LSM) to show the relationship between stress at the crack tip and stress at the far sides of the partially cracked rectangular plate. The crack was in the form of a continuous single line and was located at the centre of the plate. Israr et al. [4] applied the line spring model and obtained the vibration characteristics of the partially cracked isotropic plate containing a single partial line crack parallel to one of the edges of the plate. They found that fundamental frequency decreases with the increase in crack length. The literature shows that the fundamental frequency of the plate is affected by crack location, orientation, length, boundary condition, internal material length scale parameter and environment. To the best of the author's knowledge, the literature lacks the results for vibration analysis of the S-FGM plate containing a partial crack. Thus, the present work addresses the following.

i) Vibration analysis of S-FGM plate with a centrally located partial crack.
ii) The effect of crack length on vibration characteristics of cracked S-FGM plate
iii) The effect of gradient index on cracked S-FGM plate vibration characteristics.

Results and Discussion

The results are presented for a square FGM plate, and the sigmoid function defines the volume fractions. The dimension of the square S-FGM plate is considered as L1/h=100. The length of crack '2a' is varied from 0 to 0.1m, and the gradient index 'n' is varied from 0 to 5. The crack depth is 0.006m. 'z/h' is the normalized thickness of the plate. The results are presented for all side simply supported boundary condition (SSSS). It is observed that as the crack length increases the fundamental frequency reduces. Further, it is also observed that the fundamental frequency increases with the increase in z/h ratio for all the case undertaken. In the case of Power law distribution, it was observed that the fundamental frequency increases with the increase in gradient index 'n' but in the case of S-FGM plate it is found that the fundamental frequency remains same for all the values of gradient index when 'z/h=0'.

References

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